Perturbative Infra-red physics of Yang-Mills theories

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ABSTRACT

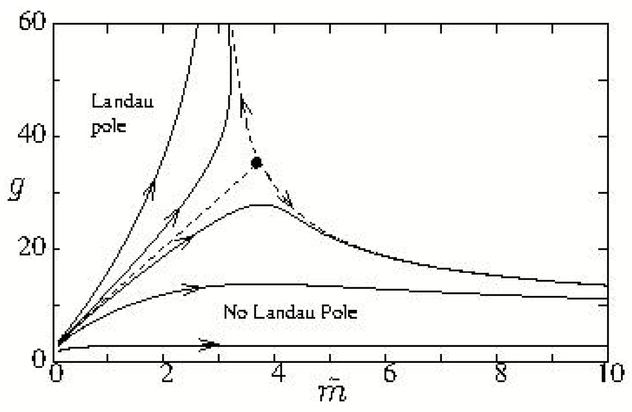
We investigate the infra-red properties of Yang-Mills correlators in a novel Gribov-ambiguity-free class of non-linear covariant gauges. These family of one-parameter gauges (indexed by ξ below) are continuously connected to the well-studied Landau gauge, and provide a non-perturbative generalization of the Curci-Ferrari-Delbourgo-Jarvis gauges, that might be amenable to lattice simulations. We explicitly compute at one-loop order in perturbation theory the gluon and ghost propagator, as well as the renormalization group flows, down to the deep infra-red regime. In particular, we show that there exists infra-red safe trajectories with no Landau pole, we find that both the gluon and ghost propagators develop a mass gap and that even in non-Landau gauge the gluon propagator is transverse.

MOTIVATION

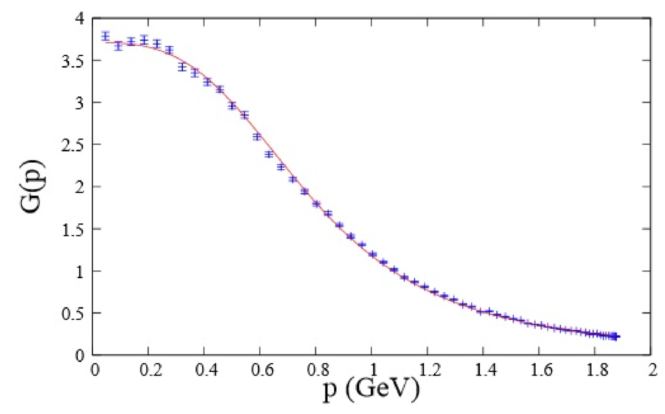
Lifting the Gribov ambiguity:
It is well known that the standard Faddeev-Popov gauge fixing procedure, used to quantize Yang-Mills theories, is plagued by the existence of Gribov copies. Recently, a novel approach was proposed in [1] based on averaging over the Gribov copies, leading to a gauge-fixed Yang-Mills action free of Gribov ambiguities and with effectively massive gluons. Firsts studies were performed in the particular case of the Landau gauge within this gauge-fixed action.

Landau gauge:

- There exists infra-red safe trajectories allowing perturbative development down to the deep infra-red regime. See left figure, $\tilde{m} = \frac{m}{\mu}$ with m the gluon mass, μ the RG scale, g the coupling constant.
- One-loop perturbative computations of ghost and gluon propagators are in remarkable agreement with lattice simulations. See right figure, $G(p)$ is the gluon propagator.



One-loop RG flow in the plane $(\tilde{m}, g)$. The arrows point toward the infra-red [1].

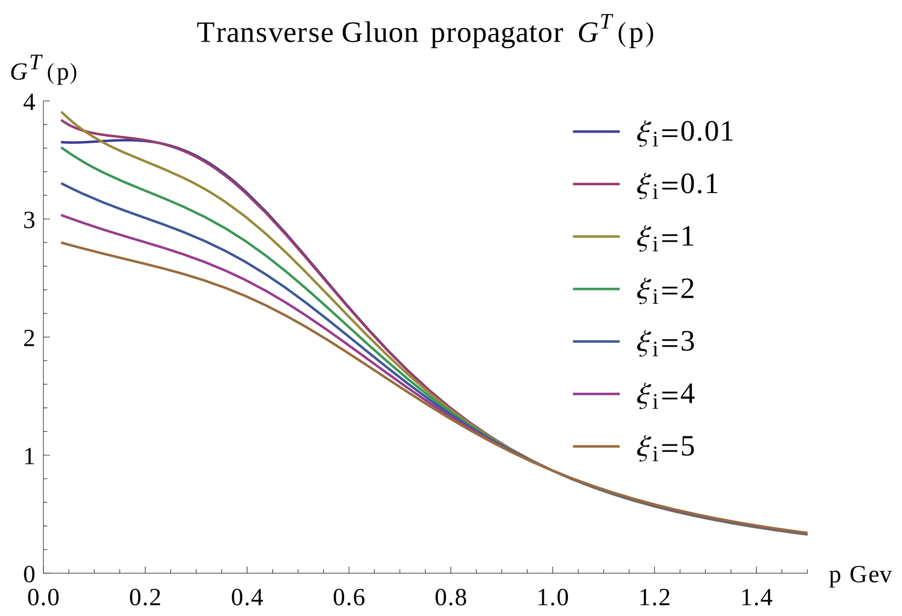


Blue points: lattice results [4].
Red curve: one-loop perturbative computations [3].

These exciting results motivate the study outside the Landau gauge in order to investigate the possible gauge dependent effects in the infra-red.

GLUON AND GHOST PROPAGATORS COMPUTED AT ONE-LOOP

We display the gluon and ghost propagators computed at one-loop order in perturbation theory with renormalization group improvement. The computations are performed within these new one-parameter, ξ, gauges (see below). We defined the RG initial conditions at $\mu_0 = 1$ Gev and we show our results for various $\xi(\mu_0) \equiv \xi_i$.

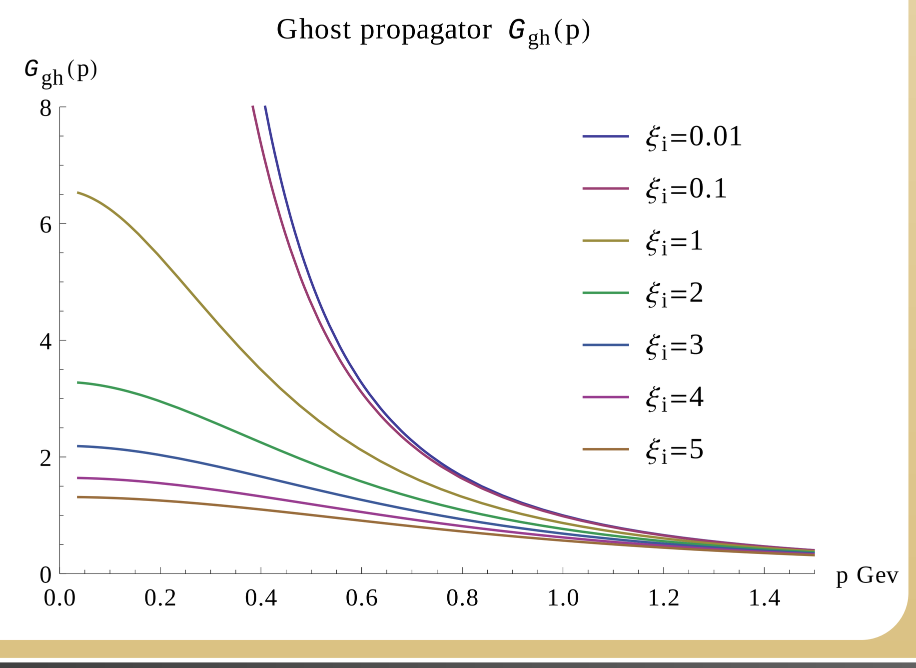


Gluon propagator:

- Transverse even in non-Landau gauge
- Massive

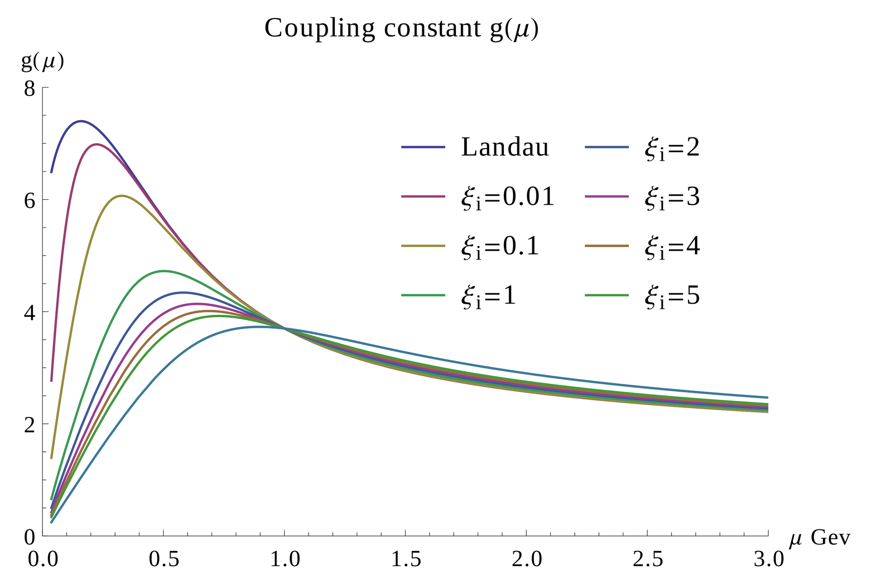
Ghosts propagator

- Develops mass gap $\propto \xi$



INFRA-RED SAFE TRAJECTORIES

RG initial conditions: $\mu_0 = 1$ Gev, $g(\mu_0) = 3.7$.



- No Landau pole

THE GAUGE FIXING PROCEDURE

Gauge condition:
In order to fix the gauge, we consider the functional

$$\mathcal{H}[A,\eta,U] = \int_x \mathrm{tr}\left[\left(A_\mu^U\right)^2 + \frac{U^\dagger\eta + \eta^\dagger U}{2}\right]$$

where η is an arbitrary $N \times N$ matrix field and A_μ^U is the gauge transform of A_μ with a gauge element $U \in \mathrm{SU}(N)$. We define our gauge condition as (one of) the extrema of $\mathcal{H}$ with respect to U.
This is a slight generalization of the Landau gauge case (recovered for $\eta = 0$, where many lattice studies are performed [5]) and therefore standard extemization techniques, e.g. the Los Alamos algorithm [6], might be applicable to the present proposal, allowing lattice studies outside the Landau gauge.

Following the standard Faddeev-Popov procedure, and averaging over η with a weight

$$\mathcal{P}[\eta] = \mathcal{N}\exp\left(-\frac{g_0^2}{4\xi_0}\int_x \mathrm{tr}\,\eta^\dagger\eta\right),$$

leads to a gauge-fixing action:

$$S_{\mathrm{CFDJ}}[A,c,\bar{c},h] = \int_x \Big\{\partial_\mu\bar{c}^a D_\mu c^a + ih^a\partial_\mu A_\mu^a + \xi_0\left[\frac{(h^a)^2}{2} - \frac{g_0}{2}f^{abc}ih^a\bar{c}^b c^c - \frac{g_0^2}{4}\left(f^{abc}\bar{c}^b c^c\right)^2\right]\Big\}$$

that corresponds to the Curci-Ferrari-Delbourgo-Jarvis gauges.
However, these gauge-fixings suffer from Gribov ambiguities due to the presence of Gribov copies that correspond to extrema $U_i \equiv U_i[A,\eta]$ of the functional $\mathcal{H}[A,\eta,U]$ for given A and η.

Lifting the Gribov ambiguities:
We define the vacuum expectation values for any operator $\mathcal{O}[A]$ from a two-steps averaging procedure: $\overline{\langle\mathcal{O}[A]\rangle}$.
$\langle\mathcal{O}[A]\rangle$ is an average over Gribov copies and over the "noise" field η:

$$\langle\mathcal{O}[A]\rangle = \frac{\int\mathcal{D}\eta\mathcal{P}[\eta]\sum_i\mathcal{O}[A^{U_i}]s(i)e^{-\beta_0\mathcal{H}[A,\eta,U_i]}}{\int\mathcal{D}\eta\mathcal{P}[\eta]\sum_i s(i)e^{-\beta_0\mathcal{H}[A,\eta,U_i]}},$$

where the sums run over all Gribov copies, $s(i)$ is the sign of the functional determinant of the Faddeev-Popov operator evaluated at $U = U_i$ and β_0 is a free parameter which controls the lifting of degeneracy between Gribov copies according to the value of the functional $\mathcal{H}[A,\eta,U_i]$.
In particular we have for a gauge invariant operator that: $\langle\mathcal{O}_{\mathrm{inv}}[A]\rangle = \mathcal{O}_{\mathrm{inv}}[A]$.

This average over the Gribov copies can be cast into a functional integral formulation

$$\langle\mathcal{O}[A]\rangle = \frac{\int\mathcal{D}\mathcal{V}\,O[A^U]\,e^{-S_{\mathrm{CF}}[A,\mathcal{V}]}}{\int\mathcal{D}\mathcal{V}\,e^{-S_{\mathrm{CF}}[A,\mathcal{V}]}}, \tag{1}$$

that involves the Curci-Ferrari action (see below), $S_{\mathrm{CF}}[A,\mathcal{V}]$.
Using a super matrix field $\mathcal{V}$

$$\mathcal{V}(x,\theta,\bar{\theta}) = \exp\left\{ig_0\left(\bar{\theta}c + \bar{c}\theta + \bar{\theta}\theta\hat{h}\right)\right\}U,$$

defined on a super space parametrized by two Grassmannian coordinates $\underline{\theta} = (\theta,\bar{\theta})$ with metric g^{MN} defined as $g^{\theta\bar{\theta}} = -g^{\bar{\theta}\theta} = \beta_0\bar{\theta}\theta - 1$, the Curci-Ferrari action takes the very compact form:

$$S_{\mathrm{CF}}[A,\mathcal{V}] = \frac{1}{g_0^2}\int_{x,\underline{\theta}}\mathrm{tr}\left\{D_\mu\mathcal{V}^\dagger D_\mu\mathcal{V} + \frac{\xi_0}{2}g^{MN}\partial_N\mathcal{V}^\dagger\partial_M\mathcal{V}\right\},$$

Replica :
Once the Gribov degeneracy has been lifted for each individual gauge field configuration, we average over the latter with the Yang-Mills weight (denoted by an overall bar):

$$\overline{\langle\mathcal{O}[A]\rangle} = \frac{\int\mathcal{D}A\,\langle\mathcal{O}[A]\rangle\,e^{-S_{\mathrm{YM}}[A]}}{\int\mathcal{D}A\,e^{-S_{\mathrm{YM}}[A]}}.$$

Note however that the denominator in Eq.(1) depends explicitly on the field configuration A, and therefore it is a non trivial task to formulate this two-steps average $\overline{\langle\mathcal{O}[A]\rangle}$ as a functional integral with one local action.
This can be efficiently done using the replica trick, that is making use of the identity

$$\frac{1}{\int\mathcal{D}\mathcal{V}\,e^{-S_{\mathrm{CF}}[A,\mathcal{V}]}} = \lim_{n\to 0}\int\prod_{k=1}^{n-1}\left(\mathcal{D}\mathcal{V}_k\,e^{-S_{\mathrm{CF}}[A,\mathcal{V}_k]}\right).$$

The resulting Lagrangian is the one we used for our perturbative computations:

$$\begin{aligned}\mathcal{L} \;=\; & \frac{1}{4}\left(F_{\mu\nu}^a\right)^2 + \partial_\mu\bar{c}^a D_\mu c^a + \beta_0\left(\frac{1}{2}(A_\mu^a)^2 + \xi_0\bar{c}^a c^a\right) + ih^a\partial_\mu A_\mu^a \\ & + \xi_0\left[\frac{(h^a)^2}{2} - \frac{g_0}{2}f^{abc}ih^a\bar{c}^b c^c - \frac{g_0^2}{4}\left(f^{abc}\bar{c}^b c^c\right)^2\right] \\ & + \underbrace{\frac{1}{g_0^2}\sum_{k=2}^{n}\int_{\underline{\theta}_k}\mathrm{tr}\left\{D_\mu\mathcal{V}_k^\dagger D_\mu\mathcal{V}_k + \frac{\xi_0}{2}g^{MN}\partial_N\mathcal{V}_k^\dagger\partial_M\mathcal{V}_k\right\}}_{\mathcal{L}_{SUSY}}.\end{aligned}$$

In particular at the bare level, each replica contributes to the gluon square mass a factor β_0 leading to an effective gluon square mass $n\beta_0$. The ghosts are effectively massive outside the Landau gauge, with a tree level square mass $\xi_0\beta_0$

RENORMALIZATION

The previous gauge-fixed Lagrangian was shown to be renormalizable to all orders of perturbation theory [2] independently of the way the limit $n \to 0$, induced by the replica, is taken.
We therefore have considered renormalization schemes in which

$$n\beta_0 = Z_{m^2}m^2 \quad \xi_0 = Z_\xi\xi_R n$$

such that the two tree level renormalized square masses survive to the $n \to 0$ limit. As a consequence, the gluon propagator remains transverse even in non-Landau gauge, and the theory presents infra-red safe renormalization group trajectories.

PERSPECTIVES

The formulation of the gauge condition derived as an extremization of the functional $\mathcal{H}$ is a slight generalization of the one routinely employed in the Landau gauge where lattice simulations are performed. We therefore expect (see [2]), that the present proposal is amenable to lattice simulations . This would provide non perturbative computations outside the Landau gauge to which we could compare our predictions.

On the other hand, our two-steps averaging procedure allows the computations of new quantities such that correlations between different gauge orbits

$$\overline{\langle A\rangle\langle A\rangle} \;=\; \overline{\langle AA\rangle} + \overline{\langle A\partial\Lambda\rangle} + \ldots$$

that involve non trivial correlators of the super-symmetric fields. We thus believe that information on Gribov copies might be encoded in the super-symmetric sector of our theory, which is currently under studies.

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