Spacetime symmetries of the Quantum Hall Effect

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SEW14

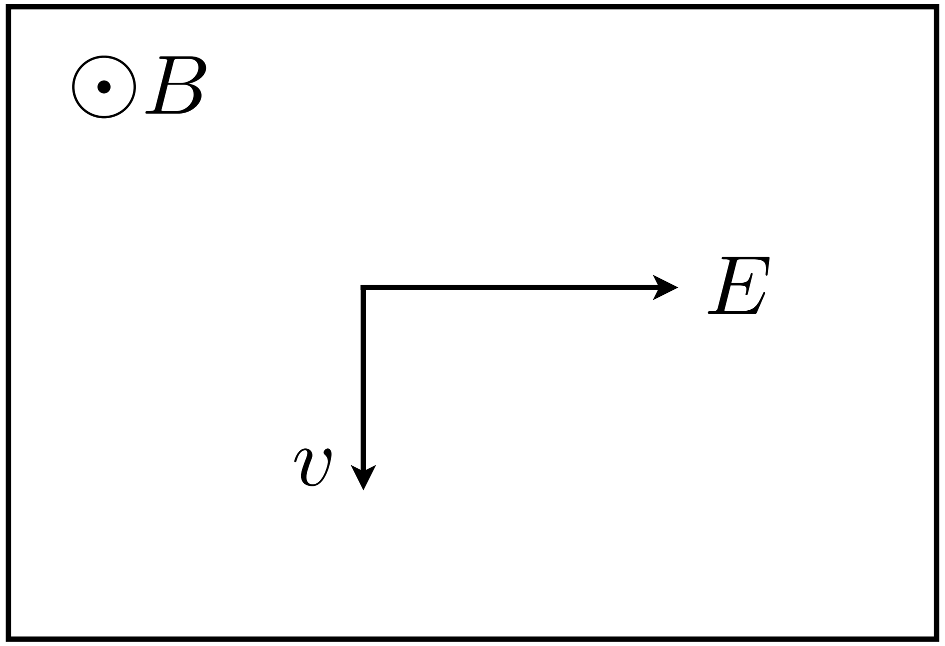
Siavash Golkar, Matt Roberts, DTS, arXiv:1403.4279

DTS arXiv:1306.0638

Plan

- Review of quantum Hall physics
- Relativistic QHE
  - new Chern-Simons term
- Symmetries
- Newton-Cartan geometry

Classical Hall effect

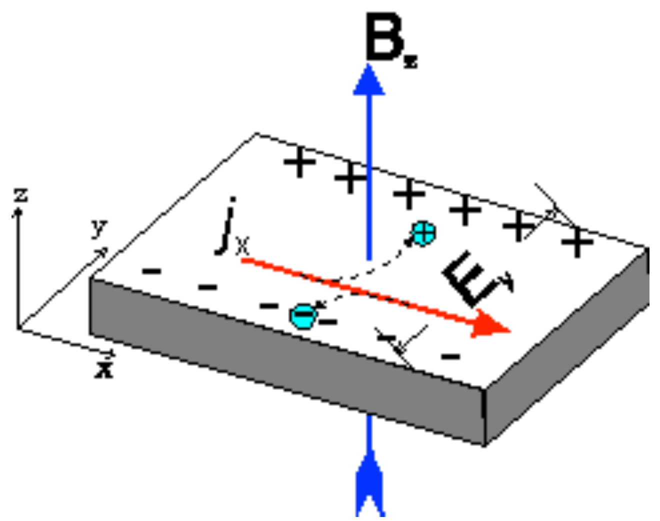


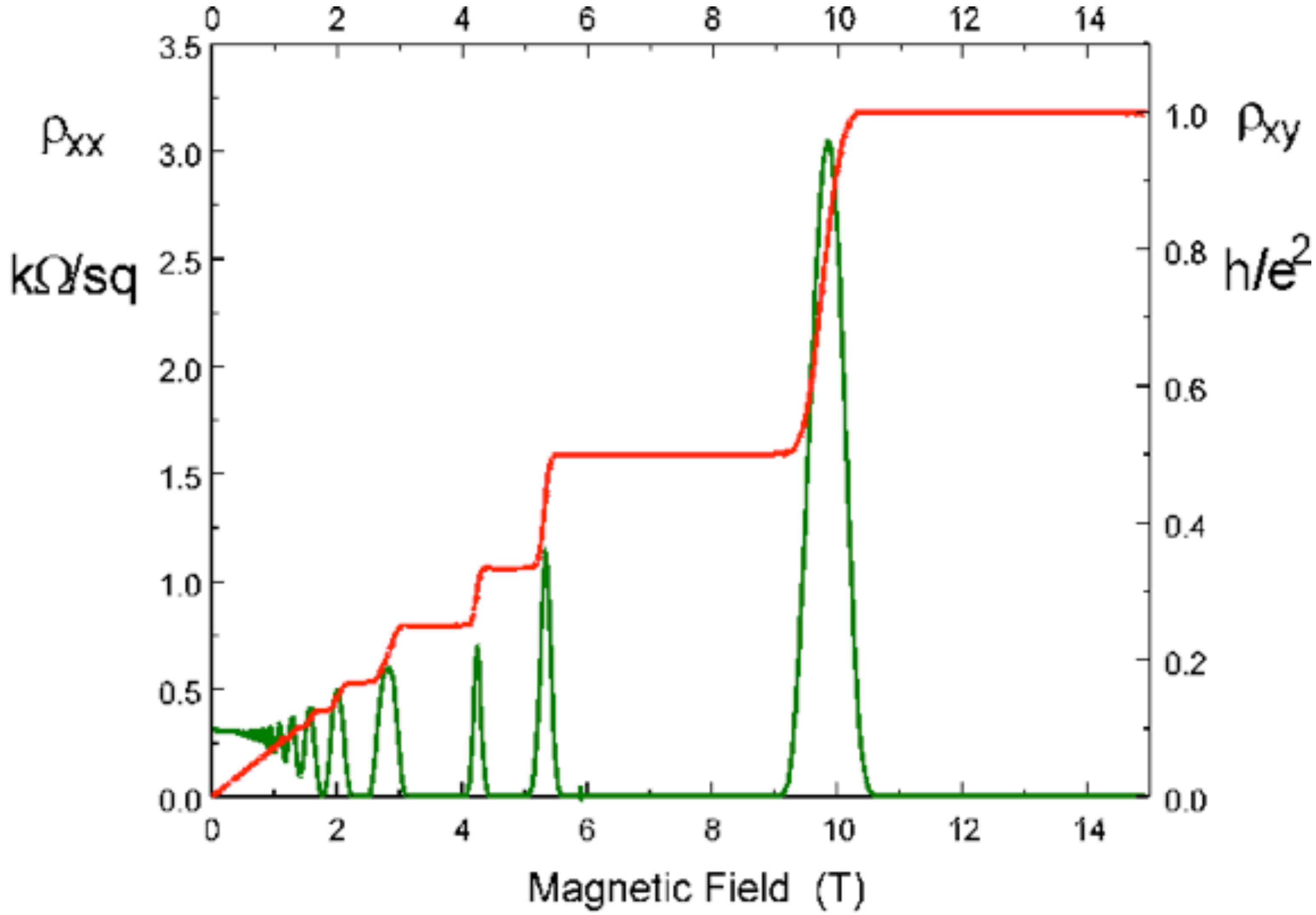
$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

$$\rho_{xy} = \frac{E_x}{j_y} = \frac{B}{e\rho}$$

2D electron density

Quantum Hall effect

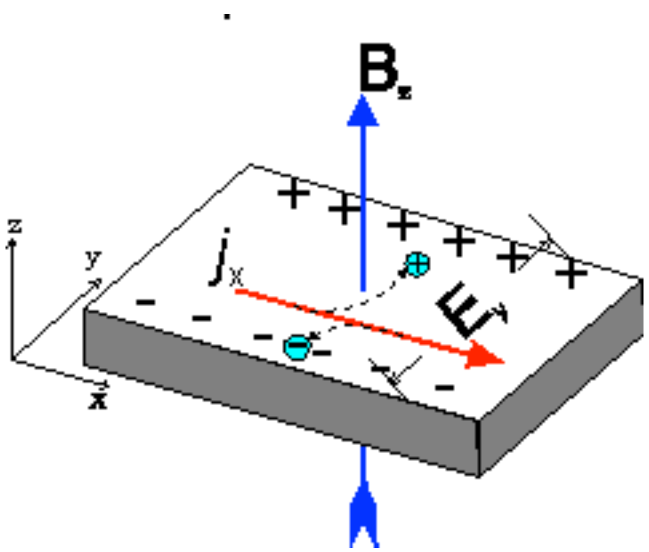


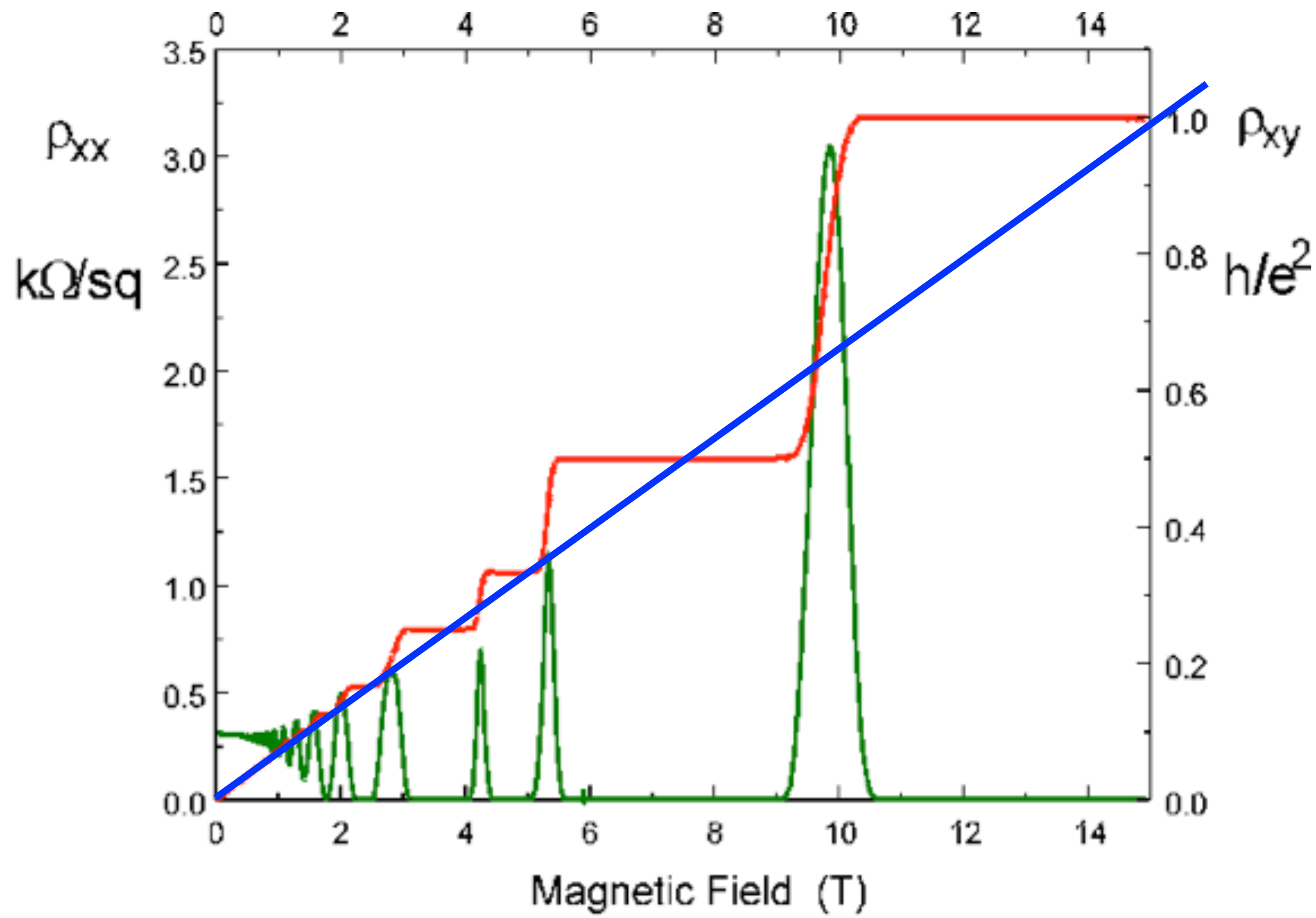


$$\sigma_{xy} = \frac{1}{\rho_{xy}} = n\frac{e^2}{2\pi\hbar}$$

integer

Quantum Hall effect



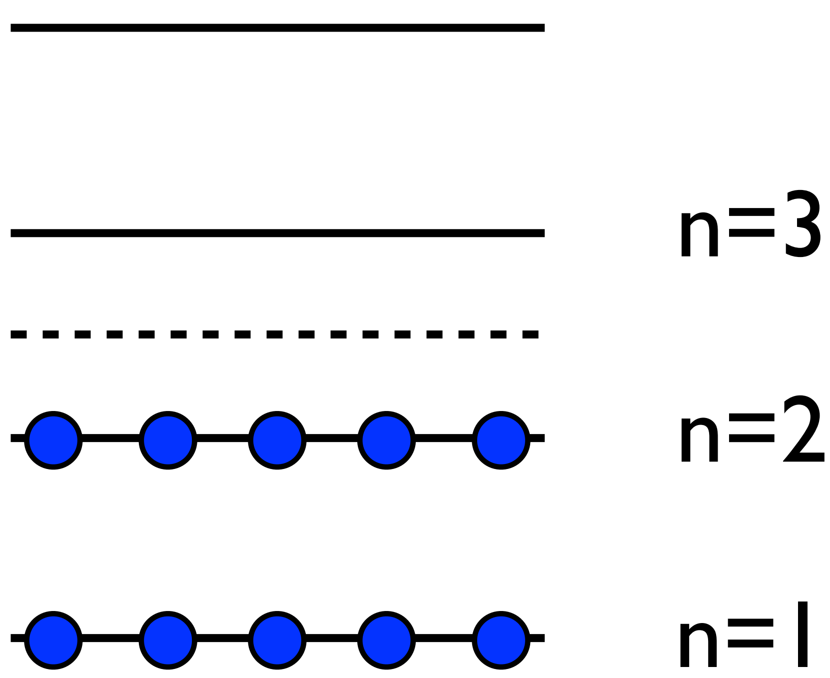


$$\sigma_{xy} = \frac{1}{\rho_{xy}} = n\frac{e^2}{2\pi\hbar}$$

integer

Integer quantum Hall state

- electrons filling n Landau levels

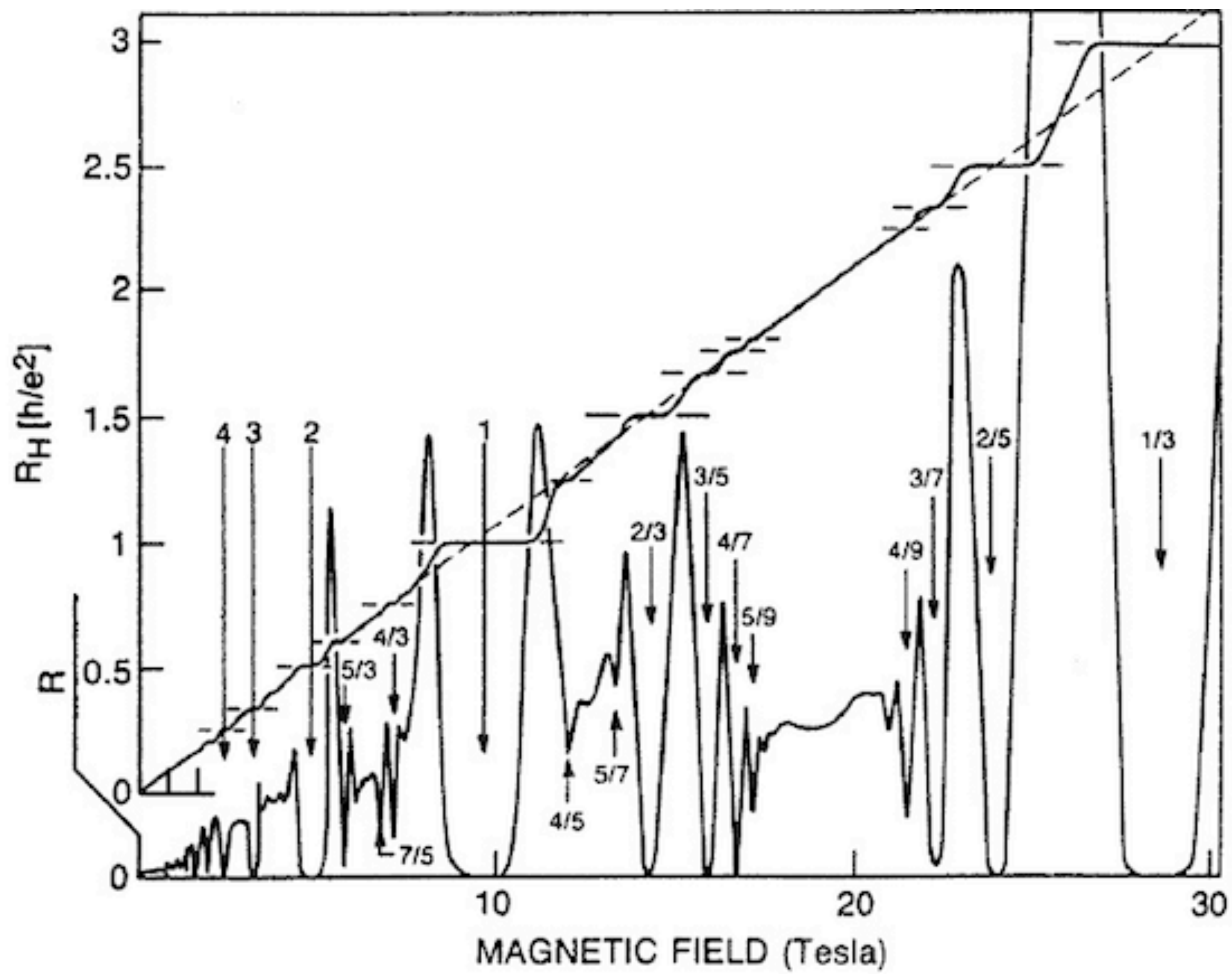


$$n_{2\mathrm{D}} = n\frac{eB}{2\pi\hbar}$$

$$\sigma_{xy} = \frac{en_{2\mathrm{D}}}{B} = n\frac{e^2}{2\pi\hbar}$$

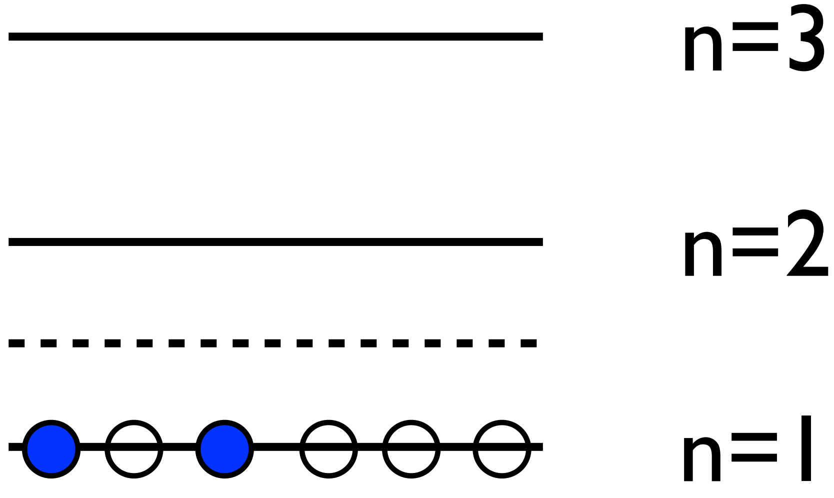
Electrons above filled Landau levels: localized by defects

Fractional QH effect



$$\sigma_{xy} = \frac{p}{q}\frac{e^2}{2\pi\hbar}$$

Fractional quantum Hall state

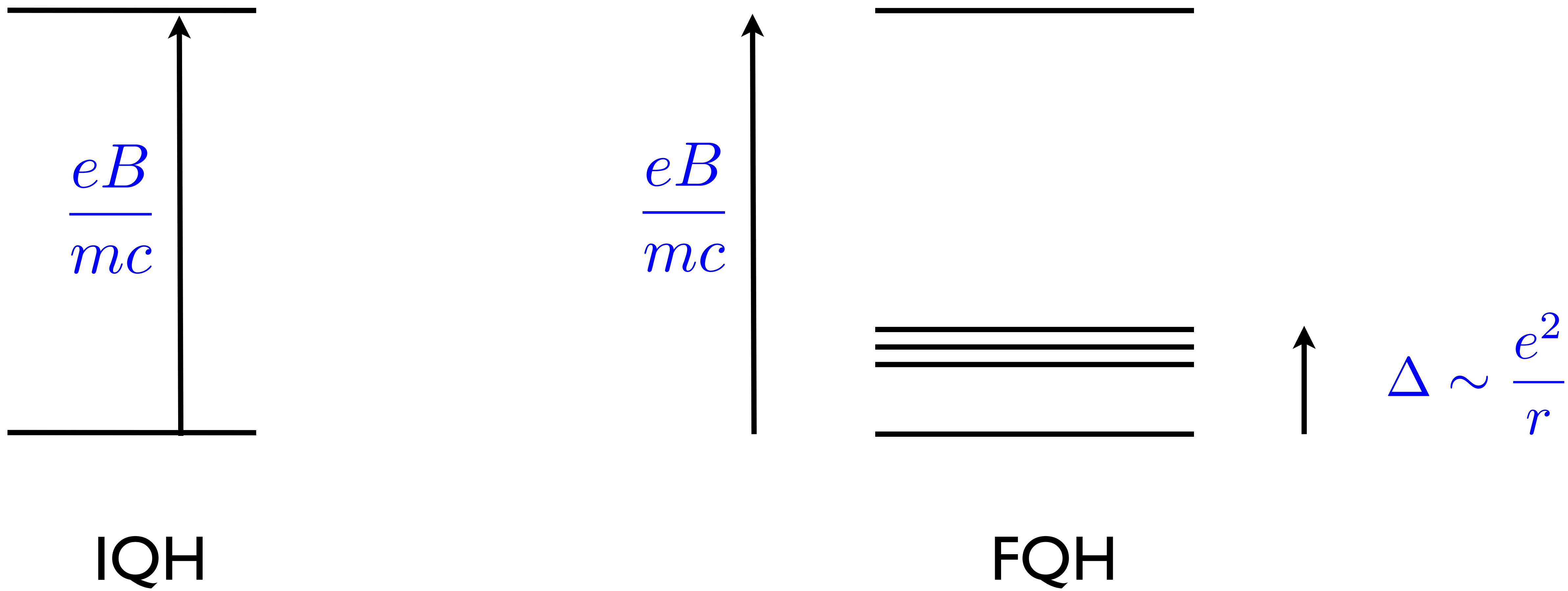


Without interactions, ground state has huge degeneracy

Interactions somehow lift the degeneracy, make system gapped at particular values of the filling factor

$$\nu = \frac{n}{B/2\pi} \qquad \nu = \frac{1}{3}, \frac{1}{5}, \cdots$$

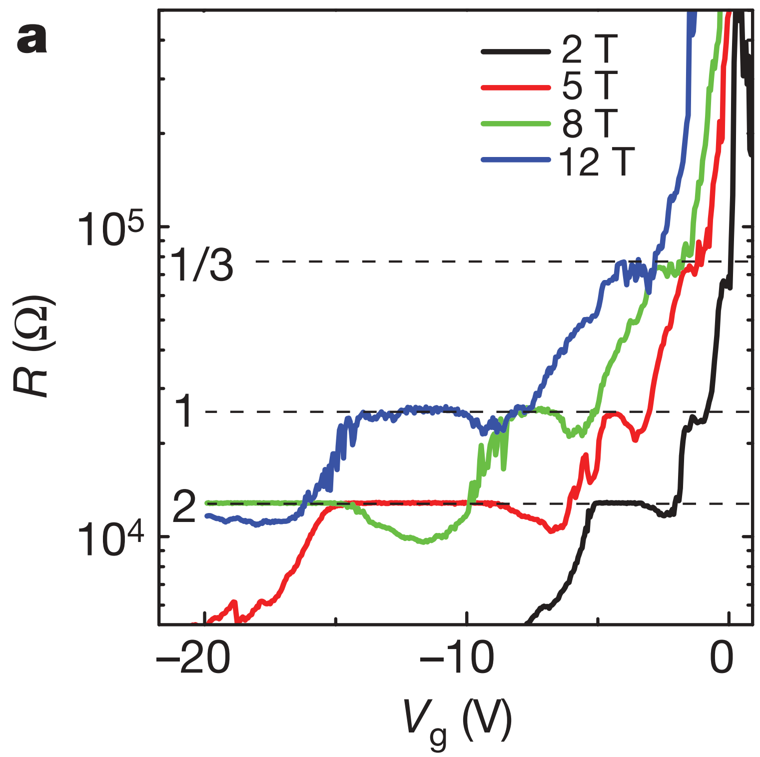
Energy scales

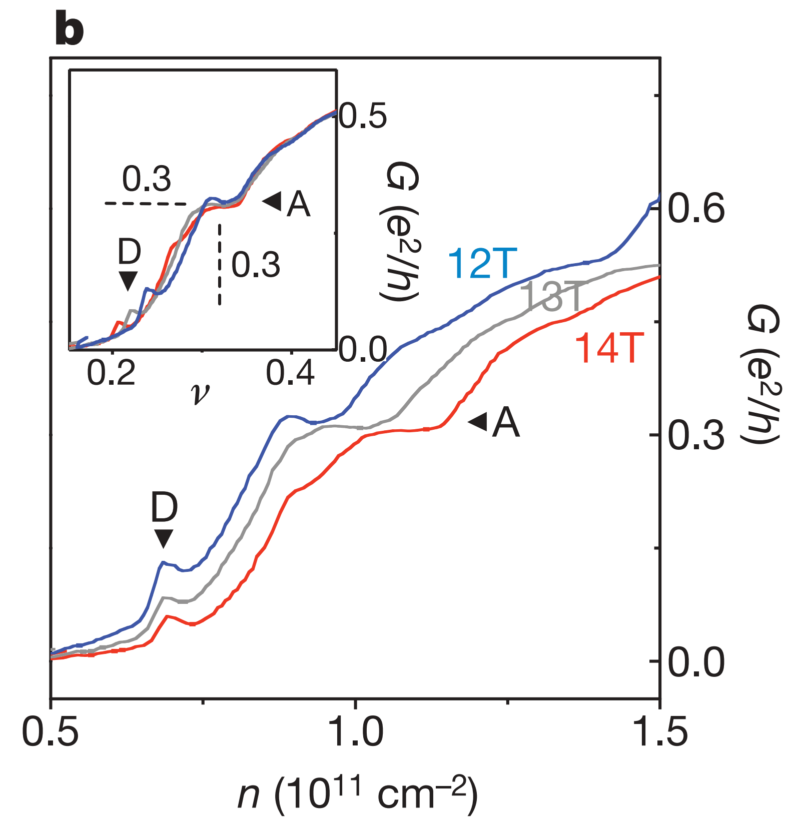


Interesting limit: eB/mc >> Δ (m→0 limit)
only lowest Landau level (LLL) states survives

No small parameter

QHE in graphene





Du et al, Nature 492, 192 (2009)

Bolotin et al, Nature 492, 192 (2009)

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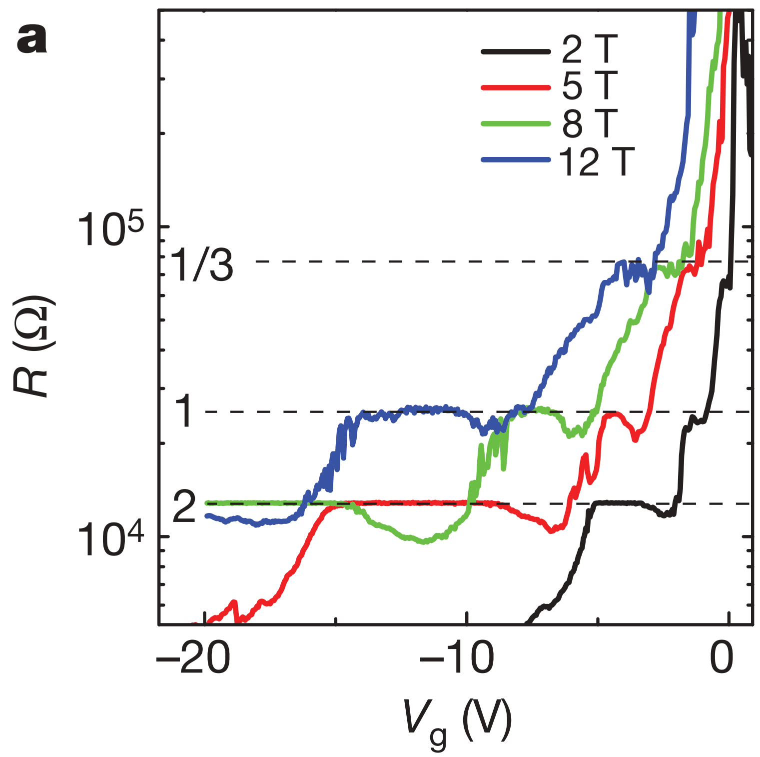

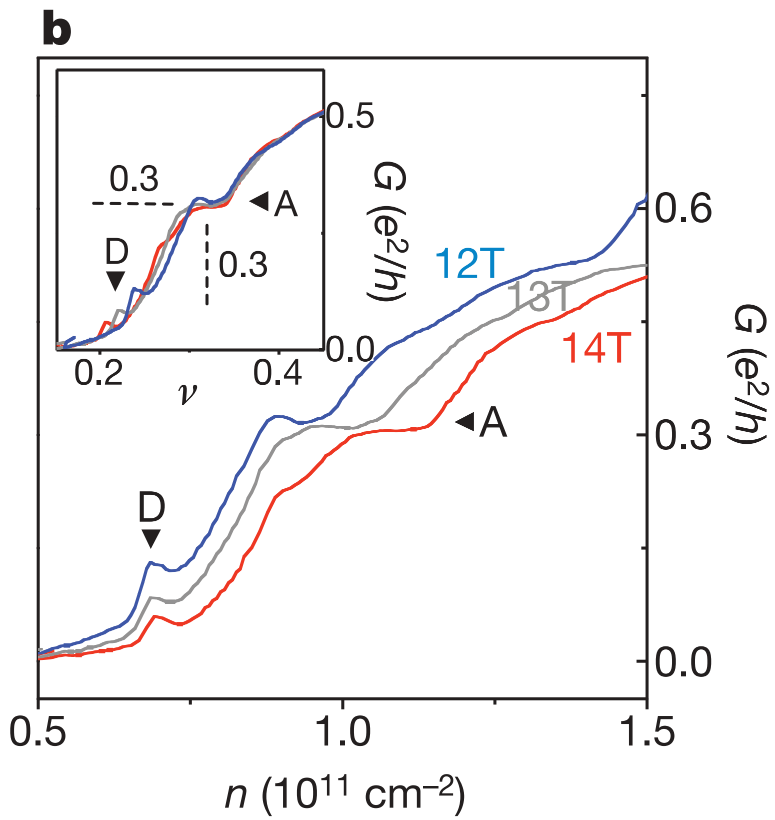

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We will use graphene as a training ground

(review of graphene: Semenoff's talk)

Relativistic QH effect

- Consider relativistic fermions in a magnetic field
- What is the low-energy effective theory of the quantum Hall states?
- gap: no low-energy degree of freedom
- local effective action

$$S = S_{\text{eff}}[A_\mu, g_{\mu\nu}]$$

Relativistic invariance

- The effective theory must be relativistically invariant

$$Z[A_\mu] = \int D\psi\, D\bar{\psi}\, \exp(iS[A_\mu, \psi, \bar{\psi}])$$

$$A_\mu \to A'_\mu \qquad\qquad S_{\text{eff}}[A_\mu] = S_{\text{eff}}[A'_\mu]$$

In the same way the effective action must be general-coordinate invariant

Power counting

- The effective action can be expanded in powers of fields and of derivatives
- To organize the expansions, we give count fields as different powers of momentum
- One possible scheme is

$$F_{\mu\nu} = O(p^0) \qquad A_\mu = O(p^{-1})$$

$$g_{\mu\nu} = O(p^0)$$

Order $O(p^{-1})$

- One term at order $O(p^{-1})$

$$S = \frac{\nu}{4\pi}\int d^3x\,\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda$$

Encodes information about Hall conductivity

$$j^\mu = \frac{\delta S}{\delta A_\mu} = \frac{\nu}{4\pi}\epsilon^{\mu\nu\lambda}F_{\nu\lambda}$$

$$J_y = \sigma_{xy}E_x \qquad \sigma_{xy} = \frac{\nu}{2\pi}\frac{e^2}{\hbar}$$

Order O(p^0)

- At order O(p^0) $F_{\mu\nu}^2$

  We can instead use b and u^μ

$$bu^\mu = \frac{1}{2}\varepsilon^{\mu\nu\lambda}F_{\nu\lambda} \qquad b = \left(\frac{1}{2}F_{\mu\nu}F^{\mu\nu}\right)^{1/2}$$

$$u^\mu u_\mu = -1$$

$$S = \cdots - \int d^3x\, \sqrt{-g}\, \epsilon(b)$$

ϵ(b): energy density as function of magnetic field

Order O(p)

- From b and u^μ it seems that the only term to order O(p) that one can form is

$$S = \cdots + \int d^3x\, f(b)\epsilon^{\mu\nu\lambda}u_\mu\partial_\nu u_\lambda$$

f(b) determines by the dynamics

But there is another term to O(p) order

Topological current

- Flat space: identically conserved current $u^\mu u_\mu = -1$

$$J^\mu = \varepsilon^{\mu\nu\lambda}\varepsilon^{\alpha\beta\gamma}\, u_\alpha\, \partial_\nu u_\beta\, \partial_\lambda u_\gamma \qquad \partial_\mu J^\mu = 0$$

In curved space

$$J^\mu = \varepsilon^{\mu\nu\lambda}\varepsilon^{\alpha\beta\gamma} u_\alpha \left(\nabla_\nu u_\beta \nabla_\lambda u_\gamma - \frac{1}{2} R_{\nu\lambda\beta\gamma} \right) \qquad \nabla_\mu J^\mu = 0$$

"Euler current"

Analogy with O(3) σ model

$$J^\mu = \epsilon^{\mu\nu\lambda}\epsilon^{abc} n^a \partial_\nu n^b \partial_\nu n^c \qquad n^a n^a = 1$$

$$Q = \frac{1}{8\pi}\int d^2x\, \epsilon^{ij}\epsilon^{abc}\, n^a \partial_i n^b \partial_j n^c$$

$S^2 \rightarrow S^2$

$$n^a \rightarrow u^\mu \qquad u^\mu u_\mu = -1$$

Second topological term

$$\int d^3x \sqrt{-g}\, \frac{\kappa}{8\pi} A_\mu J^\mu$$

$$= \frac{\kappa}{8\pi} \int d^3x \sqrt{-g}\, A_\mu \varepsilon^{\mu\nu\lambda} \varepsilon^{\alpha\beta\gamma} u_\alpha \Big(\nabla_\nu u_\beta \nabla_\lambda u_\gamma - \frac{1}{2} R_{\nu\lambda\beta\gamma} \Big)$$

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- κ has topological interpretation

Relativistic “shift”

Relativistic "shift"

$$J^0 = -\frac{1}{2}\epsilon^{0\nu\lambda}\epsilon^{\alpha\beta\gamma}u_\alpha R_{\nu\lambda\beta\gamma} + \cdots = 2R_{1212} + \cdots$$

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- Consider the QH state on a curved manifold

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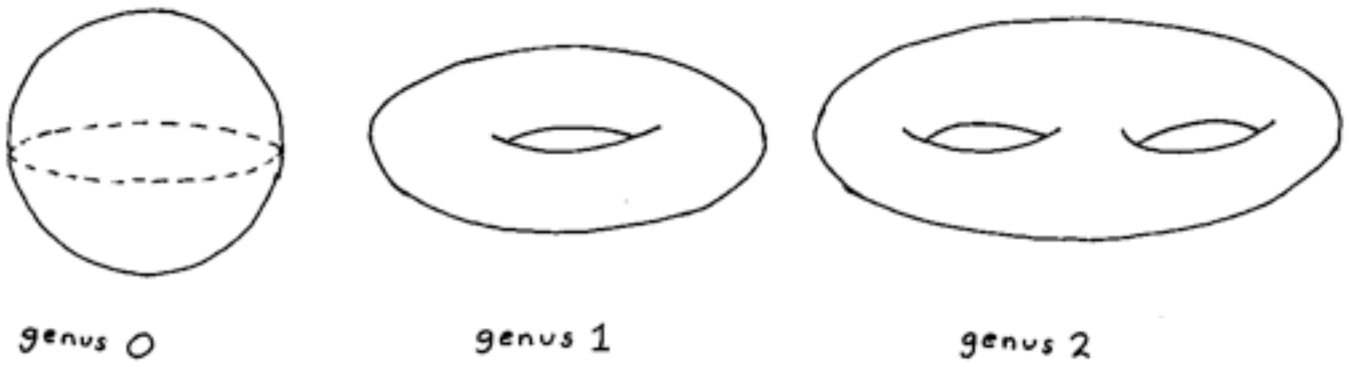

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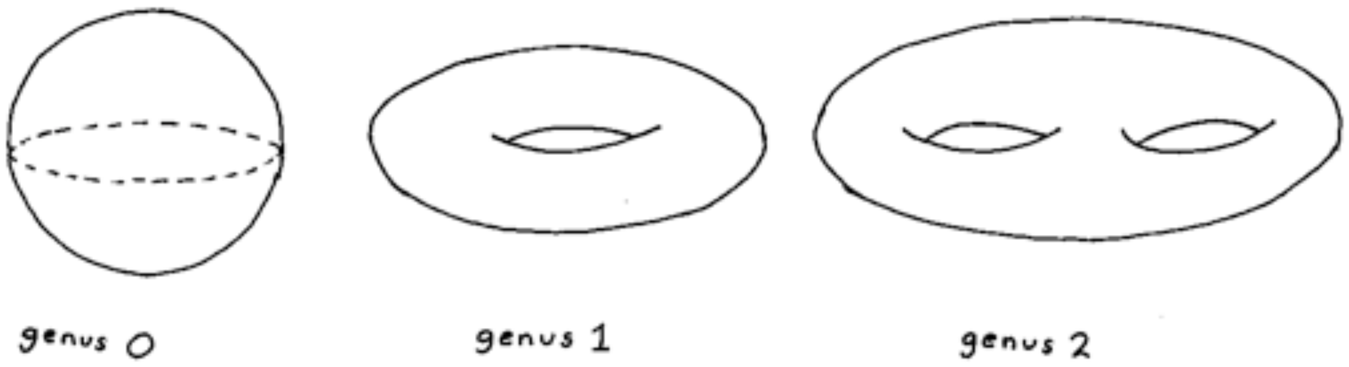

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"shift" (Wen-Zee)

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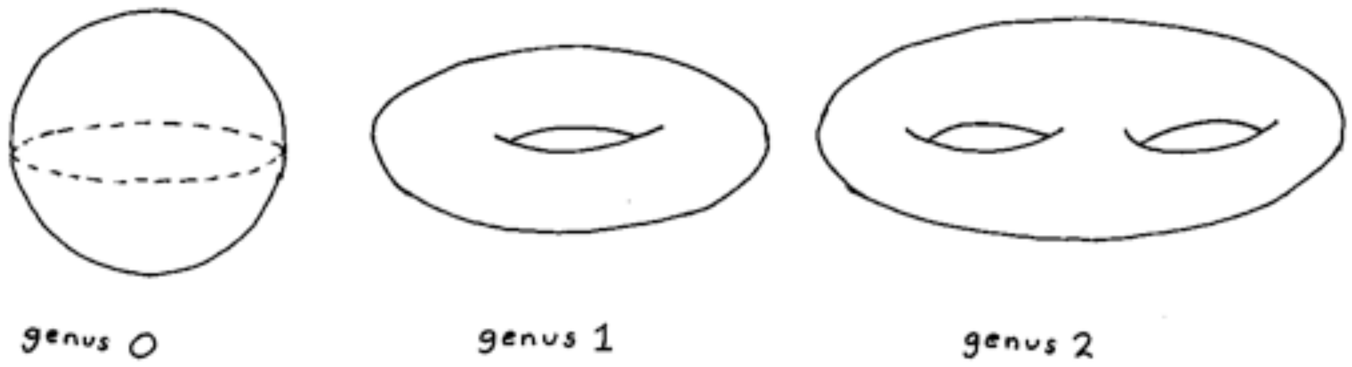

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"shift" (Wen-Zee)

for example: $\nu = 1/3$ $\kappa = 2/3$

Topology and dynamics

- Thus the coefficient of the new term is determined topologically
  - cannot change under small change of parameters
- But at the same time, the term itself is not topological (depends on the metric)
- contributes to correlation functions

Hall viscosity

- Consider metric perturbations

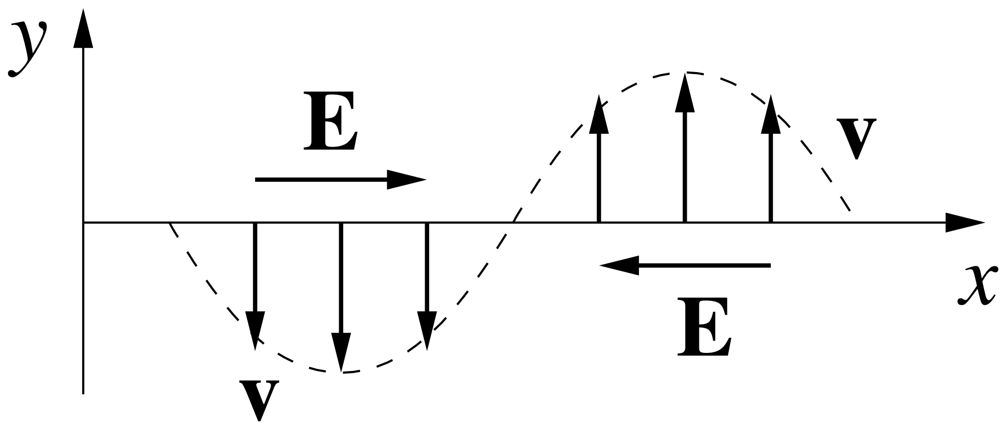
$$g_{ij} = \delta_{ij} + h_{ij}(t) \qquad h_{ii} = 0$$

$$S = -\frac{\kappa B}{32\pi} \int d^3x \, \epsilon^{jk} h_{ij} \partial_t h_{ik}$$

$$\langle T_{11} T_{12} \rangle = i\eta_{\mathrm{H}} \omega$$

$$\eta_{\mathrm{H}} = \frac{\kappa B}{8\pi}$$

Response to inhomogeneous E



$$E_x = E\,\mathrm{e}^{iqx}$$

$$j_y = \sigma_{xy}(q) E_x$$

$$\sigma_{xy}(\omega, q) = \frac{\nu}{2\pi} + \left(\frac{\kappa}{4\pi} + \frac{2f(B)}{B}\right)\frac{\omega^2}{B} - \frac{f'(B)}{B}\frac{q^2}{B}$$

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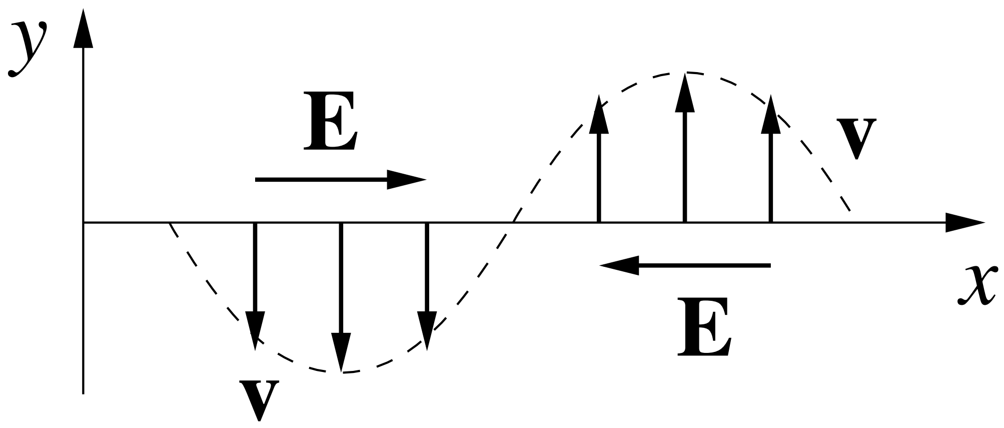

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LLL limit: $f(b) = \frac{1}{8\pi}(\nu - \kappa)b$

$$\sigma_{xy}(\omega, q) = \frac{\nu}{2\pi} + \frac{\nu}{\pi}\frac{\omega^2}{B} + \frac{\kappa - \nu}{8\pi}\frac{q^2}{B}$$

Zeroth Landau level symmetry

- For a FQH state on the zeroth Landau level

$$(\partial_{\bar z} - iA_{\bar z})\psi = 0 \qquad \begin{aligned}(D_x + iD_y)\psi = 0 \\ (D_x - iD_y)\psi^\dagger = 0\end{aligned}$$

Stress tensor in static magnetic field: $T^{\mu\nu} = -\frac{i}{4}\bar\psi\gamma^\mu \overleftrightarrow{D}^\nu \psi$

$$T^{0i} = -\frac{i}{4}(\psi^\dagger D_i\psi - D_i\psi^\dagger\psi) = -\frac{1}{4}\epsilon^{ij}\partial_j(\psi^\dagger\psi)$$

From effective action $T^{0i} = \frac{\delta S}{\delta g_{0i}} = -\epsilon^{ij}\partial_j\Big(\frac{\kappa}{8\pi}b + f(b)\Big)$

$$f(b) = \frac{1}{8\pi}(\nu - \kappa)b$$

More generally

- How much of what we learned in relativistic systems can be extended to nonrelativistic systems (GaAs)?

Nonrelativistic case

$$S = \int d^3x \, \sqrt{g} \, \left[\frac{i}{2} \psi^\dagger \overleftrightarrow{D}_t \psi - \frac{g^{ij}}{2m} D_i \psi^\dagger D_j \psi + \gamma \frac{B}{4m} \psi^\dagger \psi \right]$$

+ interactions

$D_\mu \psi \equiv (\partial_\mu - i A_\mu) \psi$

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γ=2: LLL degenerate with zero energy in any magnetic field and metric

$$H = \int d^2x \, \frac{1}{m} D_z \psi^\dagger D_{\bar{z}} \psi + O(m^0)$$

$$z = x + iy$$

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$$H = \int d^2x\, \frac{1}{m} D_z\psi^\dagger D_{\bar{z}}\psi + O(m^0)$$ $z = x + iy$

LLL limit $m \to 0$ constraint $D_{\bar{z}}\psi = 0$

correlation functions are finite

NR general coordinate inv.

DTS, M.Wingate 2006

Gauge invariance: $\psi \to e^{i\alpha}\psi$ $\quad A_\mu \to A_\mu + \partial_\mu \alpha$

General coordinate invariance:

$$\delta\psi = -\xi^k \partial_k \psi$$

$$\delta A_0 = -\xi^k \partial_k A_0 - A_k \dot{\xi}^k + \frac{1}{2}\varepsilon^{ij}\partial_i (g_{jk}\dot{\xi}^k)$$

$$\delta A_i = -\xi^k \partial_k A_i - A_k \partial_i \xi^k - m g_{ik}\dot{\xi}^k$$

$$\delta g_{ij} = -\xi^k \partial_k g_{ij} - g_{kj}\partial_i \xi^k - g_{ik}\partial_j \xi^k$$

Galilean transformations: special case $\xi^i = v^i t$

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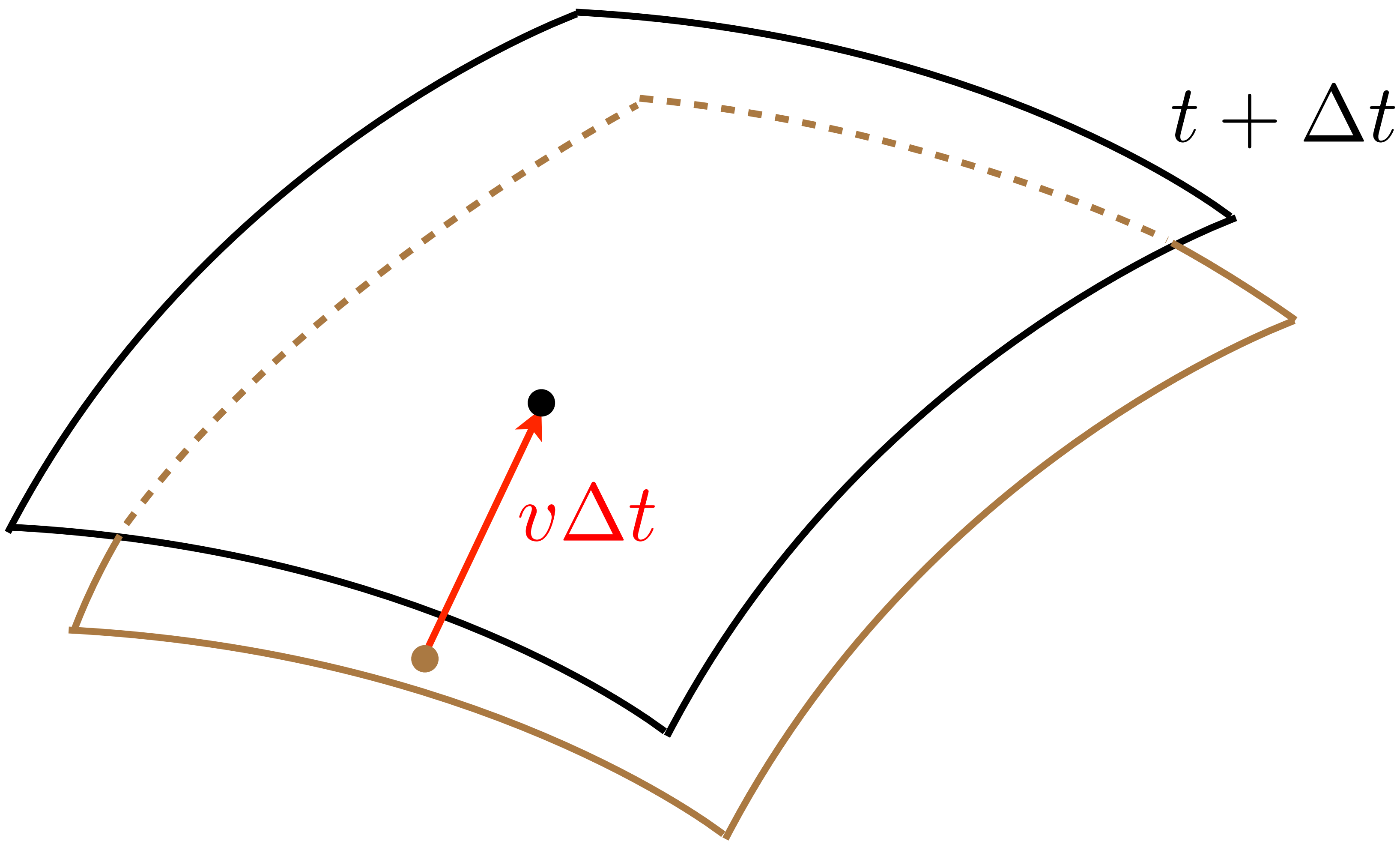
Galilean transformations: special case $\xi^i = v^i t$

Effective theory must respect these unusual symmetries

More on geometry

- System does not live in a 3D Riemann space
- 2D Riemann manifold at any time slice
  - can parallel transport along equal-time slices, but one need new information to transport between different times

Velocity vector v



A vector v needed to parallel transport objects from one time slice to another

Newton-Cartan structure: $(g_{ij},\ v^i)$

Newton-Cartan geometry

Newton-Cartan geometry

$$(g^{\mu\nu},\, n_\mu,\, v^\mu) \qquad g^{\mu\nu} n_\nu = 0 \qquad n_\mu v^\mu = 1$$

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$$(g^{\mu\nu},\, n_\mu,\, v^\mu) \qquad g^{\mu\nu} n_\nu = 0 \qquad n_\mu v^\mu = 1$$

$$dn = 0 \Rightarrow n = dt$$ choose t to be time coordinate

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$dn = 0 \Rightarrow n = dt$ choose t to be time coordinate

$$g^{\mu\nu} g_{\nu\lambda} = \delta^\mu_\lambda - v^\mu n_\lambda \qquad g_{\mu\nu} v^\nu = 0$$

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$$g^{\mu\nu} g_{\nu\lambda} = \delta^\mu_\lambda - v^\mu n_\lambda \qquad g_{\mu\nu} v^\nu = 0$$

$$g^{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & g^{ij} \end{pmatrix} \qquad g_{\mu\nu} = \begin{pmatrix} v^2 & -v_j \\ -v_i & g^{ij} \end{pmatrix}$$

Newton-Cartan connection

$$\Gamma^{\lambda}_{\mu\nu} = v^{\lambda}\partial_{\mu}n_{\nu} + \frac{1}{2}g^{\lambda\rho}(\partial_{\mu}g_{\rho\nu} + \partial_{\nu}g_{\rho\mu} - \partial_{\rho}g_{\mu\nu})$$

Properties:

$$\nabla_{\lambda}g^{\mu\nu} = 0 \qquad \nabla_{\mu}n_{\nu} = 0 \qquad g_{\alpha[\mu}\nabla_{\nu]}v^{\alpha} = 0$$

Improved gauge potentials

- With v one can construct a gauge potential that transforms as a one-form

$$\tilde{A}_i = A_i$$

$$\tilde{A}_0 = A_0 - \frac{1}{2}\varepsilon^{ij}\partial_i(g_{jk}v^k)$$

$$\delta\tilde{A}_\mu = -\xi^k\partial_k\tilde{A}_\mu - \tilde{A}_k\partial_\mu\xi^k$$

What is v?

- Microscopic Lagrangian does not involve v
- there is a freedom to choose v
- One possible choice is

$$\tilde{F}_{\mu\nu} v^{\nu} = 0$$

$$v^i = \frac{\epsilon^{ij} E_j}{B} + \cdots$$

drift velocity

Effective field theory

$$S = \int d^3x\, \epsilon^{\mu\nu\lambda} \left(\frac{\nu}{4\pi} \tilde{A}_\mu \partial_\nu \tilde{A}_\lambda + \frac{\kappa}{2\pi} \tilde{A}_\mu \partial_\nu \omega_\lambda - \frac{c}{48\pi} \omega_\mu \partial_\nu \omega_\lambda \right) + \cdots$$

$$\omega_\mu = \frac{1}{2} \epsilon^{ab} e^{a\nu} \nabla_\mu e^b_\nu$$ spin connection

↑ Newton-Cartan covariant derivative

$\kappa = \nu \mathcal{S}$ $\mathcal{S} = \mathrm{shift}$

$c =$ number of boundary modes? Abanov, Gromov 2014

Conclusions

- There is a nontrivial interplay between topology and geometry in quantum Hall effects
- Symmetries when put in curved space, implying nontrivial results in flat space
  - response to inhomogeneous EM field
- Constraints on possible holographic realizations of the FQHE

Thank you