# Polyakov-loop potential from a massive extension of the background field gauge

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## Semi-analytical approaches to strongly interacting matter



Need for semi-analytical methods to investigate infrared properties of QCD or related theories.

These approaches (SD-eq, fRG, ...) usually require gauge fixing:

Ex: Landau gauge action

$$S = \int_{X} \left\{ \frac{1}{4} F^{a}_{\mu\nu} F^{a}_{\mu\nu} + \partial_{\mu} \bar{c}^{a} (D_{\mu}c)^{a} + i h^{a} \partial_{\mu} A^{a}_{\mu} \right\}$$

Valid only in the UV where the Gribov ambiguity is expected not to play a role.

 $\Rightarrow$  Some additional imput is needed in the IR.



## Extended Landau gauge (eLG)

Alternative approach: find phenomenologically (and hopefully theoretically) motivated actions that could take into account the existence of Gribov copies.

A candidate for such an action is the extended Landau gauge (eLG) action

$$S = \int_{X} \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \partial_{\mu} \bar{c}^a (D_{\mu}c)^a + i h^a \partial_{\mu} A^a_{\mu} + \frac{1}{2} m^2 A^a_{\mu} A^a_{\mu} \right\}$$

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- It is perturbatively renormalizable.
- A perturbative, calculation of the T = 0 propagators and vertices reproduces lattice data!



Peláez, Tissier, Wschebor, Phys.Rev. D88 (2013) and arXiv:1407.2005.

 It could result from a gauge fixing procedure which averages over Gribov copies. (Serreau, Tissier, Phys.Lett. B712 (2012); Serreau, Tissier, Tresmontant, arXiv:1307.6019).

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- It is perturbatively renormalizable. Extra parameter: for SU(3),  $m \approx 510 \text{ MeV}$ .
- A perturbative, calculation of the T = 0 propagators and vertices reproduces lattice data!



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## Tests at finite temperature

One loop, finite *T*, eLG ghost and chromo-magnetic propagators agree well with lattice results:



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The eLG fails in reproducing the lattice chromo-electric propagator in the vicinity of the confinement/deconfinement phase transition:

- could signal a failure of the eLG model
  - → but similar limitations are observed in other approaches.
- could signal limitations of the use of the LG
  - → explore "more appropriate" gauges and test whether the corresponding (IR) extended gauge models are capable to describe the phase transition.

## Polyakov loop and center-symmetry breaking

Free-energy F for having an isolated static quark located somewhere

$$e^{-\beta F} = \frac{1}{N} \left( \operatorname{tr} P e^{ig \int_0^\beta d\tau A_0(\tau)} \right) \equiv \langle L \rangle \quad \text{with} \quad A_0 = A_0^a t^a \ (a = 1, \dots, N)$$

The Yang-Mills action at finite T is invariant under twisted or center (gauge) transformations

$$U(\beta, \vec{x}) = U(0, \vec{x})V \quad \text{with} \quad V \in SU(N)_{\text{center}} = \left\{ e^{i2\pi k/N} \mathbb{1} | k = 0, \dots, N-1 \right\}$$

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Under a center transformation  $\langle L \rangle \rightarrow \langle L \rangle e^{i2\pi k/N}$ :

- if center-symmetry is broken  $\langle L \rangle \neq 0$  and  $F < \infty$  (deconfined phase);
- if center-symmetry is restored  $\langle L \rangle = 0$  and  $F = \infty$  (confined phase);

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The lattice predicts a 2nd/1st order breaking of center-symmetry in the SU(2)/SU(3) case. Confirmed by the functional renormalization group:



J. Braun, H. Gies and J.M. Pawlowski, Phys.Lett. B684 (2010).

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Can this physics be captured perturbatively?

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# Extended background field gauge

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## The extended background field gauge

Choose a background  $\bar{A}^a_{\mu}$ . Fix the gauge according to  $(\bar{D}_{\mu}(A_{\mu} - \bar{A}_{\mu}))^a = 0$ . In the limit  $\xi \to 0$ , one obtains the Landau-deWitt gauge:

$$S_{\bar{A}}[A] = \int_{X} \left\{ \frac{1}{4} F^{a}_{\mu\nu} F^{a}_{\mu\nu} + (\bar{D}_{\mu}\bar{c})^{a} (D_{\mu}c)^{a} + ih^{a} (\bar{D}_{\mu}(A_{\mu} - \bar{A}_{\mu}))^{a} \right\}$$

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Why to consider such a gauge? From  $S_{\bar{A}}[A]$ , it is possible to construct  $\tilde{\Gamma}[\bar{A}]$  such that

- the physics is obtained at the absolute minimum of  $\tilde{\Gamma}[\bar{A}]$ ;
- center-symmetry is manifest because  $\tilde{\Gamma}[\bar{A}^U] = \tilde{\Gamma}[\bar{A}]$ .

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We upgrade the bakground field gauge to the extended background field gauge (eBFG):

$$S_{\bar{A}}[A] = \int_{X} \left\{ \frac{1}{4} F^{a}_{\mu\nu} F^{a}_{\mu\nu} + (\bar{D}_{\mu}\bar{c})^{a} (D_{\mu}c)^{a} + ih^{a} (\bar{D}_{\mu}(A_{\mu} - \bar{A}_{\mu}))^{a} + \frac{1}{2} m^{2} (A^{a}_{\mu} - \bar{A}^{a}_{\mu}) (A^{a}_{\mu} - \bar{A}^{a}_{\mu}) \right\}$$

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The mass term does not break center symmetry!

Feynman rules?

## Feynman rules: simplifying remarks

We are interested in thermodynamical properties:

- $\Rightarrow$  uniform background:  $\bar{A}^{a}_{\mu}(\tau, \vec{x}) = \bar{A}^{a}_{\mu}$ .
- $\Rightarrow$  effective potential:  $\gamma(\bar{A}) = \tilde{\Gamma}[\bar{A}]/(\beta V)$ .

We are interested in the Polyakov loop:

 $\Rightarrow$  temporal background  $\bar{A}^a_{\mu} = \bar{A}^a_0 \delta_{\mu 0}$ .

One can always choose  $\bar{A}_0$  in the Cartan sub-algebra:

$$\Rightarrow SU(2): \overline{A}_0 = \overline{A}_0^3 \frac{\sigma^3}{2}$$
  
$$\Rightarrow SU(3): \overline{A}_0 = \overline{A}_0^3 \frac{\lambda^3}{2} + \overline{A}_0^8 \frac{\lambda^6}{2}$$
  
...

The only role of the background is to lift the usual degeneracy between the three color directions.

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## Feynman rules: modes

Ex.: SU(2) ghost propagator:

SU(2) gluon propagator:

eLG:

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## Feynman rules: modes

Ex.: SU(2) ghost propagator:

SU(2) gluon propagator:

eLG: 
$$\begin{cases} G_{\mu\nu}^{11}(K) = \frac{P_{\mu\nu}^{1}(K)}{K^{2} + m^{2}} + \frac{\xi P_{\mu\nu}^{\parallel}(K)}{K^{2} + \xi m^{2}} \\ G_{\mu\nu}^{22}(K) = G_{\mu\nu}^{11}(K) \rightarrow eBFG: \\ G_{\mu\nu}^{33}(K) = G_{\mu\nu}^{11}(K) \end{cases} \xrightarrow{\rightarrow} eBFG: \begin{cases} G_{\mu\nu}^{0}(K) = \frac{P_{\mu\nu}^{1}(K)}{K^{2} + m^{2}} + \frac{\xi P_{\mu\nu}^{\parallel}(K)}{K^{2} + \xi m^{2}} \\ G_{\mu\nu}^{-}(K) = \frac{P_{\mu\nu}^{1}(K)}{K^{2} + m^{2}} + \frac{\xi P_{\mu\nu}^{\parallel}(K)}{K^{2} + \xi m^{2}} \end{cases}$$

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For each charge eigenstate, we have:

3 massive transverse gluons, 1 massless longitudinal gluon ( $\xi \rightarrow 0$ ), 2 massless ghosts.

 $\mathbf{P}^{\perp}(\mathbf{K}) \subset \mathbf{P}^{\parallel}(\mathbf{K})$ 

# Polyakov-loop potential and center-symmetry breaking

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## Background as an order parameter

To discuss center-symmetry breaking from  $\gamma(\bar{A})$ , it is first necessary to identify  $\bar{A}$  as an order parameter for center-symmetry breaking.

At LO, the path ordering in  $\langle L \rangle$  does not play a role

$$\langle L \rangle = \frac{1}{N} \left( \operatorname{tr} P \, e^{-ig \int_0^\beta d\tau \, (\bar{A}_0 + a_0(\tau))} \right) = \underbrace{\frac{1}{N} \operatorname{tr} e^{-i\beta g \bar{A}_0}}_{\equiv \langle L \rangle_{\text{lo}}} + \mathcal{O}(g^2)$$

**SU(2)**:  $\bar{A}_0 = \bar{A}_0^3 \frac{\sigma^3}{2}$ 

$$\langle L \rangle_{\rm lo} = \cos\left(\frac{\beta g \bar{A}_0^3}{2}\right) \Rightarrow \left[ \langle L \rangle_{\rm lo} = 0 \quad \text{iff} \quad r_3 \equiv \beta g \bar{A}_0^3 = \pi [2\pi] \right]$$

The background plays the role of an order parameter for center-symmetry breaking!

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Only the charged modes contribute to the background dependence of the potential

$$\gamma(\bar{A}_0^3) = 3T \int_q \ln\left(1 + e^{-2\beta\varepsilon_q} - 2e^{-\beta\varepsilon_q}\cos(\frac{\beta g\bar{A}_0^3}{\sum_{q \in Q}})\right) - T \int_q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right)$$

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Symmetries:

Center symmetry:  $\gamma(r_3) = \gamma(r_3 + 2\pi)$  $\Rightarrow$  we can restrict to  $[0, 2\pi]$ 

Center + C-symmetry:  $\gamma(\pi + \delta r_3) = \gamma(-\pi - \delta r_3) = \gamma(\pi - \delta r_3)$  $\Rightarrow \begin{cases} \text{we can restrict to } r_3 \in [0, \pi] \\ 0 \text{ and } \pi \text{ are extrema} \end{cases}$ 



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Only the charged modes contribute to the background dependence of the potential

$$\gamma(\bar{A}_0^3) = 3T \int_q \ln\left(1 + e^{-2\beta\varepsilon_q} - 2e^{-\beta\varepsilon_q}\cos(\frac{\beta g\bar{A}_0^3}{\varepsilon_q})\right) - T \int_q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) = \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) + \frac{1}{\varepsilon_q} \int_Q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3$$

Thermal asymptotic behavior:

$$T \gg m, \underbrace{2T \int_{q} \ln(1 + e^{-2\beta q} - e^{-\beta q} \cos(r_{3}))}_{\equiv \gamma_{\text{Weiss}}(r_{3})}$$
$$T \ll m, -T \int_{q} \ln(1 + e^{-2\beta q} - e^{-\beta q} \cos(r_{3}))$$

$$=-\frac{1}{2}\gamma_{\text{Weiss}}(r_3)$$

$$\gamma_{\text{Weiss}}(r_3) = \frac{(r_3 - \pi)^4}{24\pi^2} - \frac{(r_3 - \pi)^2}{12} + \frac{7\pi^2}{360}$$



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Thermal asymptotic behavior:

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$$\gamma_{\text{Weiss}}(r_3) = \frac{(r_3 - \pi)^4}{24\pi^2} - \frac{(r_3 - \pi)^2}{12} + \frac{7\pi^2}{360}$$



reverted Weiss potential! (ghost dominate at low T; as in the fRG approach)

Only the charged modes contribute to the background dependence of the potential

$$\gamma(\bar{A}_0^3) = 3T \int_q \ln\left(1 + e^{-2\beta\varepsilon_q} - 2e^{-\beta\varepsilon_q}\cos(\frac{\beta g\bar{A}_0^3}{\varepsilon_q})\right) - T \int_q \ln\left(1 + e^{-2\beta q} - e^{-\beta q}\cos(\beta g\bar{A}_0^3)\right) = \frac{1}{\varepsilon_q}$$

Thermal asymptotic behavior:

$$T \gg m, \underbrace{2T \int_{q} \ln(1 + e^{-2\beta q} - e^{-\beta q} \cos(r_{3}))}_{\equiv \gamma_{\text{Weiss}}(r_{3})}$$

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$$\gamma_{\text{Weiss}}(r_3) = \frac{(r_3 - \pi)^4}{24\pi^2} - \frac{(r_3 - \pi)^2}{12} + \frac{7\pi^2}{360}$$



2nd order phase transition!

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We obtain a mildly first order phase transition in agreement with lattice or fRG results.



We obtain  $T_c/m \simeq 0.363$  and since  $m \simeq 510$  MeV, we obtain  $T_c \simeq 185$  MeV. Still far from the lattice ( $T_c \simeq 295$  MeV) or from fRG results ( $T_c \simeq 284$  MeV).

## LO artifacts

The Polyakov loop reaches its limiting value at a finite temperature  $T_a/T_c = 1.5$ :



Similar conclusion for SU(3) again with  $T_a/T_c = 1.38$ :



Additional singularity in thermodynamical observables in the range  $[T_c, 2T_c]$ .

## Next-to-leading order results

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## **NLO Polyakov-loop potential**



(with background-dependent propagators and background-dependent derivative vertices)

## Summary of NLO results

 $\rightarrow$  At NLO,  $\bar{A}$  plays the role of an order parameter. We find  $\langle L \rangle_{nlo} = (1 + ag^2 \beta m) \times \langle L \rangle_{lo}$  with

$$a = \frac{3}{32\pi} + \sin^2\left(\frac{r_3}{2}\right) \int \frac{d^3q}{(2\pi m)^3} \left[\frac{1}{\cosh(\beta q) - \cos(r_3)} - \frac{q^2}{\varepsilon_q^2} \frac{1}{\cosh(\beta \varepsilon_q) - \cos(r_3)}\right]$$

Since a > 0, it follows that  $\langle L \rangle_{nlo} = 0$  iff  $\langle L \rangle_{lo} = 0$  iff  $r_3 = \pi [2\pi]$ .

→ The NLO Polyakov loop potential is UV finite.

- $\rightarrow$  Our "predictions" concerning the orders of the SU(2)/SU(3) transitions remain the same.
- $\rightarrow$  We obtain improved values for  $T_c$  in the SU(3) case:

	order	LO	NLO	FRG*	Lattice**
SU(3)	1st	185 MeV	256 MeV (prelim.)	284 MeV	295 MeV
* Braun et. al, Phys.Lett. B684 (2010)			** Aouane et. al, Phys.Rev. D85 (2012).		

→ The LO artifact seems to be lifted or at least pushed to temperatures above  $3T_c$ : we do not find additional thermodynamical singularities in the range [ $T_c$ ,  $3T_c$ ].

## **Conclusions and Outlook**

- A perturbative one-loop calculation of the Polyakov-loop potential within the extended BFG allows to capture the physics of center-symmetry breaking.
- Our approach allows for a systematic determination of higher order corrections.
- Two-loop corrections are important to reach a value of the transition temperature comparable to that obtained on the lattice or with an fRG approach and to get rid of certain artifacts of the one-loop calculation.

\* \* \* \* \*

- eBFG propagators (in progress).
- Include quarks and chemical potential (in progress).
- ...
- Solid theoretical justification of extended massive gauges?

• Thermodynamics: meaningful (monotonically increasing) pressure?



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## (more) Propaganda

And please, visit the posters by:

- Marcela Peláez;
- Gergely Markó;
- Andréas Tresmontant.

## eLG from the perspective of the eBFG

The loop expansion in the eLG looks like an expansion around an instable point.



$$p = -\gamma(r_{\min})$$
  
eLG:  $r_3 = 0$  (max)  
eBFG:  $r_3 = \pi$  (min)

eLG for  $T \ll m$ :

$$p = T^4 \int_q \ln\left(1 - e^{-q}\right) + T^4 \int_q \ln\left(1 + e^{-2q} - 2e^{-q}\cos(r_3)\right) = 3T^4 \int_q \ln\left(1 - e^{-q}\right) < 0$$

eBFG for  $T \ll m$ :

$$\rho = T^{4} \int_{q} \ln \left(1 - e^{-q}\right) + T^{4} \int_{q} \ln \left(1 + e^{-q}\right)^{2} = -\frac{3}{4} T^{4} \int_{q} \ln \left(1 - e^{-q}\right) > 0$$

Effective change of nature of the degrees of freedom in the presence of the background!

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Each charged mode contributes as in the SU(2) case but with its own  $Q_3$  and  $Q_8$  charges:

$$\gamma_{su(3)}(r_3, r_8) = \gamma_{su(2)}(r_3) + \gamma_{su(2)}\left(\frac{r_3 + r_8\sqrt{3}}{2}\right) + \gamma_{su(2)}\left(\frac{-r_3 + r_8\sqrt{3}}{2}\right)$$

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