## A Hybrid Strong/Weak Coupling Approach to Jet Quenching in Strongly Coupled Plasma

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Chesler, KR 1402.6756; and Casalderrey-Solana, Gulhan, Milhano, Pablos, KR 1405.3864

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### Some Jet Quenching Questions

- How can a jet plowing through strongly coupled quarkgluon plasma lose a decent fraction of its energy and still emerge looking pretty much like an ordinary jet?
- Partial answer: if "lost" energy ends up as soft particles with momenta  $\sim \pi T$  with directions (almost) uncorrelated with jet direction. Eg more, or hotter, or moving, plasma. Natural expectation in a strongly coupled plasma...
- Still, how do the jets themselves emerge from the strongly coupled plasma looking so similar to vacuum jets?
- Best way to answer this question: a hybrid approach to jet quenching. Treat hard physics with pQCD and energy loss as at strong coupling, see what happens, for example to jet fragmentation functions, and compare to data.
- But, what is dE/dx for a "parton" in the strongly coupled QGP in  $\mathcal{N} = 4$  SYM theory? And, while we are at it, what do "jets" in that theory look like when they emerge from the strongly coupled plasma of that theory?

### What happens to 'lost' energy?

- In any strongly coupled approach, 'lost' energy is initially hydrodynamic modes with wave vector < or  $\lesssim \pi T$ .
- The attenuation distance for sound with wave vector  $\boldsymbol{q}$  is

$$x_{\text{damping}}^{\text{sound}} = v^{\text{sound}} \frac{1}{q^2} \frac{3Ts}{4\eta}$$

which means that for  $q \sim \pi T$  (or  $q \sim \pi T/2$ ) and  $v^{\text{sound}} \sim 1/\sqrt{3}$  and  $\eta/s \sim 2/4\pi$  we have

$$x_{\text{damping}}^{\text{sound}} \sim \frac{0.3}{T} \left( \text{or} \sim \frac{1.2}{T} \right) \,.$$

• Energy lost more than a few  $x_{damping}^{sound}$  before the jet emerges will thermalize, becoming soft particles in random directions. Only energy lost within a few  $x_{damping}^{sound}$  before the jet emerges will persist as sound waves moving in roughly the same direction as the jet, resulting in a pile of soft particles around the jet. Easier to see in lower *T* plasma?

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### **One More Question**

- So, why did I write "jets" instead of jets? Which is to say, what is a jet in  $\mathcal{N} = 4$  SYM theory, anyway? There is no one answer, because hard processes in  $\mathcal{N} = 4$  SYM theory don't make jets. Hatta, Iancu, Mueller; Hofman, Maldacena.
- The formation of (two) highly virtual partons (say from a virtual photon) and the hard part of the fragmentation of those partons into jets are all weakly coupled phenomena, well described by pQCD.
- Nevertheless, different theorists have come up with different "jets" in  $\mathcal{N} = 4$  SYM theory, namely proxies that share some features of jets in QCD, and have then studied the quenching of these "jets".
- For example, Chesler, Ho and KR (arXiv:1111.1691) made a collimated gluon beam, and watched it get quenched by the strongly coupled plasma. Qualitative lessons, including about stopping length, but no quantitative calculation of energy loss.

### What have we (PC+KR) done?

- We take a highly boosted light quark (Gubser et al; Chesler et al; 2008) and shoot it through a slab of strongly coupled plasma. (G and C et al computed the stopping distance for such "jets" in infinite plasma. Arnold and Vaman did same for differently constructed "jets".)
- We do the AdS/CFT version of the brick problem. (As usual, brick of plasma is not a hydrodynamic solution.)
- Focus on what comes out on the other side of the brick. How much energy does it have? How does the answer to that question change if you increase the thickness of the brick from x to x + dx? That's dE/dx.
- Yes, what goes into the brick is a "jet", not a pQCD jet. But, we can nevertheless look carefully at what comes out on the other side of the brick and compare it carefully to the "jet" that went in.
- Along the way, we will get a fully geometric characterization of energy loss. Which is to say a new form of intuition.

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A light quark "jet", incident with  $E_{in}$ , shoots through a slab of strongly coupled  $\mathcal{N} = 4$  SYM plasma, temperature T, thickness  $L\pi T = 10$ , assumed  $\gg 1$ . What comes out the other side? A "jet" with  $E_{out} \sim 0.64E_{in}$ ; just like a vacuum "jet" with that lower energy, and a broader opening angle.

And, the entire calculation of energy loss is geometric! Energy propagates along the blue curves, which are null geodesics in the bulk. Some of them fall into the horizon; that's energy loss. Some of them make it out the other side. Geometric optics intuition for *why* what comes out on the other side looks the way it does, so similar to what went in.

Chesler, Rajagopal, 1402.6756



Here, a light quark 'jet' produced next to the slab of plasma with incident energy  $E_{in} = 87\sqrt{\lambda}\pi T \sim 87\sqrt{\lambda}$  GeV shoots through the slab and emerges with  $E_{out} \sim 66\sqrt{\lambda}$  GeV. Again, the "jet" that emerges looks like a vacuum "jet" with that energy.

Geometric understanding of jet quenching is completed via a holographic calculation of the string energy density along a particular blue geodesic, showing it to be  $\propto 1/\sqrt{\sigma - \sigma_{endpoint}}$ , with  $\sigma$  the initial downward angle of that geodesic. Immediately implies Bragg peak (maximal energy loss rate as the last energy is lost). Also, opening angle of "jet"  $\leftrightarrow$  downward angle of string endpoint.



Shape of outgoing "jet" is the same as incoming "jet", except broader in angle and less total energy.

We have computed the energy flow infinitely far downstream from the slab, as a function of the angle  $\theta$  relative to the "jet" direction.



Blue curve is angular shape of the "jet" that emerges from the slab after having been quenched.

Red dashed curve is shape of vacuum "jet", in the absence of any plasma, with  $\theta$  axis stretched by some factor f (outgoing "jet" is broader in angle) and the vertical axis compressed by more than  $f^2$  (outgoing "jet" has lost energy).

After rescaling, look at how similar the shapes of the incident and quenched "jets" are!

Chesler, Rajagopal, 1402.6756 300  $\frac{1}{E_{\rm in}} \frac{dP_{\rm out}}{d\cos\theta}$ 250  $\frac{1}{E_{\rm in}} \frac{dP_{\rm in}}{d\cos\theta}$  (rescaled) 0.8  $E_{
m out}/E_{
m in}$  9.0 200 150 100 0.2 50 0 0 0.05 0.15 0.2 0.25 0.1 0.2 0.4 0.6 0.8  $L/x_{\rm stop}$ θ

We compute  $E_{out}$  analytically, by integrating the power at infinity over angle or by integrating the energy density of the string that emerges from the slab. Geometric derivation of analytic expression for  $dE_{out}/dL$ , including the Bragg peak:

$$\frac{1}{E_{\text{in}}}\frac{dE_{\text{out}}}{dL} = -\frac{4L^2}{\pi x_{\text{stop}}^2}\frac{1}{\sqrt{x_{\text{stop}}^2 - L^2}}$$

where  $\pi T x_{stop} \propto (E_{in}/(\sqrt{\lambda}\pi T))^{1/3}$ . (Not a power law in *L*,  $E_{in}$ , or *T*; it has a Bragg peak.)

One more necessary input to our hybrid approach:  $dE_{out}/dL$  for a gluon "jet". Use the fact (Chesler et al, 2008) that a gluon "jet" with energy E is like 2 quark "jets" each with energy E/2, where both the 2's are the large- $N_c$  value of  $C_A/C_F$ . So, for gluon "jets":

$$\frac{1}{E_{\text{in}}}\frac{dE_{\text{out}}}{dL} = -\frac{4L^2}{\pi x_{\text{stop}}^2}\frac{1}{\sqrt{x_{\text{stop}}^2 - L^2}}$$

where

$$x_{\text{stop}}^{\text{gluon}} = \left(\frac{C_F}{C_A}\right)^{1/3} x_{\text{stop}}^{\text{quark}}$$
 .

Note: gluon stopping length is less different from quark stopping length than weak coupling intuition suggests. This has implications for energy loss at LHC relative to that at RHIC.

### What to do next?

- A hybrid approach in which the dE/dx derived above is applied to every parton in a PYTHIA shower. Using PYTHIA to describe the aspects of jet quenching that should be described by pQCD, but assuming that the energy loss of each QCD parton in the shower is as derived above.
- Alternatively, try modelling an entire QCD jet as a "jet" ...



- Additional loss in rungs  $\rightarrow$  strongly coupled, non-perturbative
- Assign a lifetime  $\tau_f = 2 \frac{E}{Q^2}$  to every rung. Final partons fly until critical temperature is reached
- Embed hard collision into hydrodynamic plasma with  $180 < T_c < 200$  MeV Bazazov et al, 0903.4379 Hirano et al, 1012.3955
- We don't hadronize in order to keep model assumptions minimal; therefore consider jet observables only (we checked we have little sensitivity on  $Q_0$ )



(as explained in Krishna Rajagopal's talk)

Rather intrincated path length dependence with a Bragg-like peak

$$\frac{1}{E_i}\frac{dE}{dx} = -\frac{4x^2}{\pi x_{stop}^2 \sqrt{x_{stop}^2 - x^2}} \qquad x_{stop} = \frac{E_i^{1/3}}{2T^{4/3}\kappa_{SC}}$$
Gluons get a smaller stopping distance according to  $\kappa_{SC}^G = \kappa_{SC}^Q \left(\frac{C_A}{C_B}\right)^{\frac{1}{3}}$ 

10

4

## Perturbative benchmarks

• To understand the predictivity of our strongly coupled model



Not aimed at superseding more sophisticated computations











Mild disagreement towards peripheral bins may indicate the importance of quenching in the hadron gas phase

## RHIC vs LHC



# With the same value of the fit parameter we get reasonable results for RHIC as well as for LHC

Our model agrees with RHIC jet data that we have seen so far, eg on charged-jet  $R_{AA}$ . We look forward to further comparisons











#### Similar trend in all models

 $A_J \equiv \frac{p_{T,1} - p_{T,2}}{p_{T,1} + p_{T,2}}$ Dijet Asymmetry



Similar trend in all models

### Number of Photons with Associated Jet



Photon-Jet Imbalance

 $D_T^{jet}$ 

 $x_{J\gamma} \equiv$ 



Not equally normalised due to different number of photon+jet pairs



### Number of Jets with Associated Jet



Could be used to constrain the free parameter

### Jet $I_{AA}$



### Jet $I_{AA}$



## Fragmentation Functions Ratio



Data requires mild modification of fragmentation functions with respect to vacuum

Fragmentation Functions Ratio



Very soft region highly sensitive to background subtraction

Fragmentation Functions Ratio



Very soft region highly sensitive to background subtraction

Fragmentation Functions Ratio



## Fragmentation Functions Ratio

anti- $k_T$ , R = 0.3







Simultaneous description of several data sets, including centrality dependence, after fitting only one parameter



## Significance of extracted parameters

Success of models depends on the freedom to choose the fitting parameter

	Strong Coupling	Radiative	Collisional
Parameter	$0.29 < \kappa_{\rm sc} < 0.41$	$1.1 < \kappa_{\rm rad} < 2.3$	$3.1 < \kappa_{ m coll} < 5.9$

For Perturbative Benchmarks

Either the strong coupling constant is large (non-perturbative regime)

or

the kinematical logarithms are large (resummation needed)

Casalderrey-Solana and Wang, 0705.1352 Blaizot and Mehtar-Tani, 1403.2323

For Strong Coupling  $1.2 \lesssim \kappa_{SC} \mathcal{N}^{=4} \lesssim 1.6$  (not robust)

 $x_{stop}$  in QCD plasma is three or four times longer than in  $\mathcal{N} = 4$  plasma, as expected due to fewer degrees of freedom at same T

#### A Hybrid Weak+Strong Coupling Approach to Jet Quenching

Casalderrey-Solana, Gulhan, Milhano, Pablos, Rajagopal, arXiv:1405.3864

- Upon fitting one parameter, *lots* of data described well, within current error bars. Value of the fitted parameter?  $x_{stop}$  is 3 to 4 times longer in QCD plasma than in  $\mathcal{N} = 4$  SYM plasma at same T. This is not unreasonable. After all, the two theories have different degrees of freedom. Take all dependences of dE/dx from the strongly coupled calculation, but not the purely numerical factor.
- Jet quenching *looks like* perturbative fragmentation plus strongly coupled energy loss. Could it *be* that?
- All this success poses a critical question: if jet quenching observables see the liquid as a liquid, how *can* we see the pointlike quasiparticles at short distance scales? This is a prerequisite to understanding *how* a strongly coupled liquid can arise in an asymptotically free gauge theory.

### The Jet Quenching Challenge

- How can we use jets to resolve the short distance structure of the liquid? Jet quenching phenomena involve physics over a range of scales, so jet quenching has long been seen as providing such a microscope. But, how?
- In this context, the long list of successful comparisons between jet data and the predictions of the hybrid model represent something of a disappointment!
- The hybrid is a hybrid of weakly coupled vacuum physics and strongly coupled energy loss + medium physics. To the extent that such an approach describes data, that data may be used to learn about the physics of the plasma on length scales at which it is strongly coupled but it cannot tell us about the weakly coupled medium physics.
- The most interesting uses of a hybrid model of the type I have presented could in the end be the study of where it fails. (More sophisticated hybrids could be developed.)

#### A Hybrid Weak+Strong Coupling Approach to Jet Quenching

Casalderrey-Solana, Gulhan, Milhano, Pablos, Rajagopal, arXiv:1405.3864

- We need further, more discriminating, observables.
- We need more precisely measured observables, to tighten the determination of the one free parameter in the hybrid model, tightening all the colored bands in all the plots.
- And, we need to add "transverse momentum broadening", since jet quenching is not only about energy loss... And since the microscope we are looking for may be more easily found in the physics of transverse kicks than in the physics of parton energy loss ...

### What to do next?

- Alternatively, try modelling an entire QCD jet as a "jet" ...
- From this perspective, next priority is quantitative analysis of broadening of the "jets".
- How to characterize opening angle of the "jet"? Maybe  $\theta_{jet} \equiv m_{jet}/E_{jet} \equiv \sqrt{E_{jet}^2 p_{jet}^2}/E_{jet}$ ? But we have the whole profile and can compare to jet shape observables.
- QCD predicts the distribution of  $m_{in}$  (eg  $\theta_{in}$ ) for each  $E_{in}$ .  $\mathcal{N} = 4$  SYM does not; each must be specified separately. Send an ensemble of "jets", with  $\theta_{in}$  for each  $E_{in}$  distributed as in QCD, through the brick of plasma. For each "jet",  $E_{out} < E_{in}$  and  $m_{out} > m_{in}$ . Analyze distribution of  $m_{out}$  (eg  $\theta_{out}$ ) for a given  $E_{out}$ . How similar is the distribution of  $m_{out}$  for "jets" with a given  $E_{out}$  to the distribution of  $m_{in}$  for incident "jets" with energy  $E_{out}$ ?
- Can experimentalists measure change in shape of jets in PbPb collisions relative to shape of jets with the same *initial* energy in pp collisions?

### What to do next?

- Can we tailor the energy density along the dual string by hand so as to design the angular shape of the "jets" to match the angular shape of QCD jets?
- Redo the PC+KR analysis for a "jet" shooting through a hydrodynamic solution, or a disk-disk collision, rather than through a static slab.
- It is important to pursue these investigations of "jet" quenching that assume that *all* the physics is strongly coupled, to see where they lead. But, as I said at the beginning, data seem to demand a hybrid approach ...
- To advance the hybrid analysis, need more precisely measured observables, more discriminating observables, and then can add further physics to the hybrid model. Eg. transverse momentum broadening. Eg. modification of branching probabilities.

#### Gauge/String Duality, Hot QCD and Heavy Ion Collisions

Casalderrey-Solana, Liu, Mateos, Rajagopal, Wiedemann

A 460 page book, available from Cambridge University Press.

Intro to heavy ion collisions and to hot QCD, including on the lattice. Intro to string theory and gauge/string duality. Including a 'duality toolkit'.

Holographic calculations that have yielded insights into strongly coupled plasma and heavy ion collisions. Hydrodynamics and transport coefficients. Thermodynamics and susceptibilities. Far-from-equilibrium dynamics and hydrodynamization. Jet quenching. Heavy quarks. Quarkonia. Some calculations done textbook style. In other cases just results. In all cases the focus is on qualitative lessons for heavy ion physics. Heavy ion collision experiments recreating the quark–gluon plasma that filled the microseconds-old universe have established that it is a nearly perfect liquid that flows with such minimal dissipation that it cannot be seen as made of particles. String theory provides a powerful toolbox for studying matter with such properties.

This book provides a comprehensive introduction to gauge/string duality and its applications to the study of the thermal and transport properties of quark–gluon plasma, the dynamics of how it forms, the hydrodynamics of how it flows, and its response to probes including jets and quarkonium mesons.

Calculations are discussed in the context of data from RHIC and LHC and results from finite temperature lattice QCD. The book is an ideal reference for students and researchers in string theory, quantum field theory, quantum many-body physics, heavy ion physics, and lattice QCD.

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Cover illustration: an artist's impression of the hot matter produced by a heavy ion collision falling into the black hole that provides its dual description. Created by Mathias Zwygart and inspired by an image, courtesy of the ALICE Collaboration and CERN.

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Gauge/String Duality, Hot QCD and Heavy Ion Collisions Casalderrey-Solana, Liu, Mateos, Rajagopal and Wiedemann

## Gauge/String Duality, Hot QCD and Heavy Ion Collisions

Jorge Casalderrey-Solana, Hong Liu, David Mateos, Krishna Rajagopal and Urs Achim Wiedemann

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### **Beyond Quasiparticles**

- QGP at RHIC & LHC, unitary Fermi "gas", gauge theory plasmas with holographic descriptions are all strongly coupled fluids with no apparent quasiparticles.
- In QGP, with  $\eta/s$  as small as it is, there can be no 'transport peak', meaning no self-consistent description in terms of quark- and gluon-quasiparticles. [Q.p. description self consistent if  $\tau_{qp} \sim (5\eta/s)(1/T) \gg 1/T$ .]
- Other "fluids" with no quasiparticle description include: the "strange metals" (including high- $T_c$  superconductors above  $T_c$ ); quantum spin liquids; matter at quantum critical points;... *The* grand challenges at the frontiers of condensed matter physics today.
- Strongly coupled plasma with a holographic description gives us an arena in which we can obtain reliable, qualitative, insights into the behavior of matter in which quasiparticles have disappeared. But, these liquids are liquids on *all* length scales and QGP is not...

### **The Challenge**

- How can we clarify the understanding of fluids without quasiparticles, whose nature is a central mystery in so many areas of science?
- We have one big advantage: our strongly coupled liquid is not a liquid if you resolve its structure at short length scales. It is described by an asymptotically free gauge theory. Hence, at short enough length scales it *is* weakly coupled quark and gluon quasiparticles.
- One set of goals for the field is quantifying the properties and dynamics of Liquid QGP at it's natural length scales, where it has no quasiparticles.
- We must also probe, quantify and understand Liquid QGP at short distance scales, where it is made of quark and gluon quasiparticles? See how the strongly coupled fluid emerges from well-understood quasiparticles at short distances. We need a microscope.

### From N = 4 SYM to QCD

- Two theories differ on various axes. But, their plasmas are *much* more similar than their vacua. Neither is supersymmetric. Neither confines or breaks chiral symmetry.
- $\mathcal{N} = 4$  SYM is conformal. QCD thermodynamics is reasonably conformal for  $2T_c \leq T < ?$ . In model studies, adding the degree of nonconformality seen in QCD thermodynamics to  $\mathcal{N} = 4$  SYM has no effect on  $\eta/s$  and little effect on observables like those this talk.
- The fact that the calculations in  $\mathcal{N} = 4$  SYM are done at strong coupling is a feature, not a bug.
- Is the fact that the calculations in  $\mathcal{N} = 4$  SYM are done at  $1/N_c^2 = 0$  rather than 1/9 a bug??
- In QCD thermodynamics, fundamentals are as important as adjoints. No fundamentals in  $\mathcal{N} = 4$  SYM, and so far they have only been added as perturbations. This, and  $1/N_c^2 = 0$ , are in my view the biggest reasons why our goals must at present be limited to qualitative insights.